

Reconstrução tomográfica a partir de projeções

EPUSP/PEE-5892

UNIFESP/PIS-110

Sérgio S Furuie

❑ Reflexão

- Acústica: Ultra-som, Radar, ..
- Ótica: microscópio confocal

❑ A partir das projeções

- No domínio do espaço
 - transmissão : CT
 - emissão: SPECT, PET
- No domínio da frequência
 - Ressonância Magnética (geometric projection, Fourier projection)

Tomografia a partir de projeções no espaço

❑ Conceito

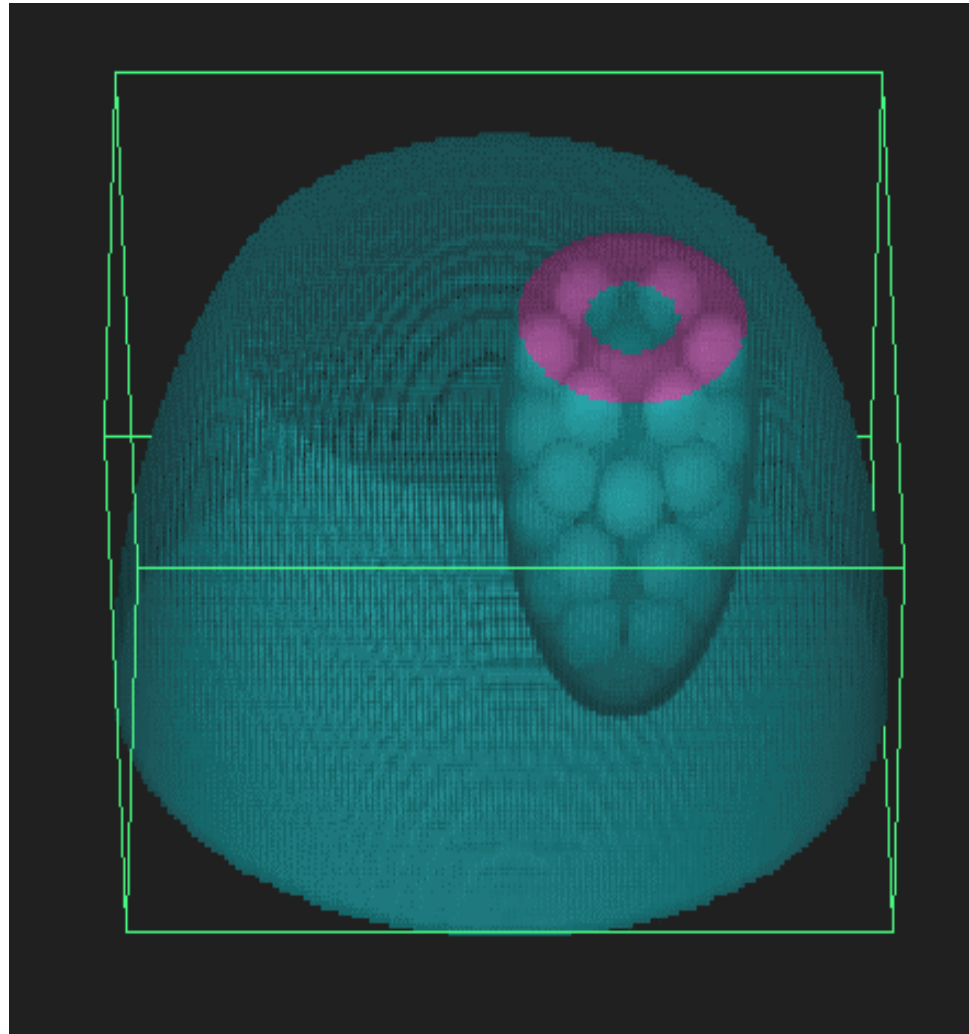
- Matemática da reconstrução: Radon, 1917

❑ Aplicação

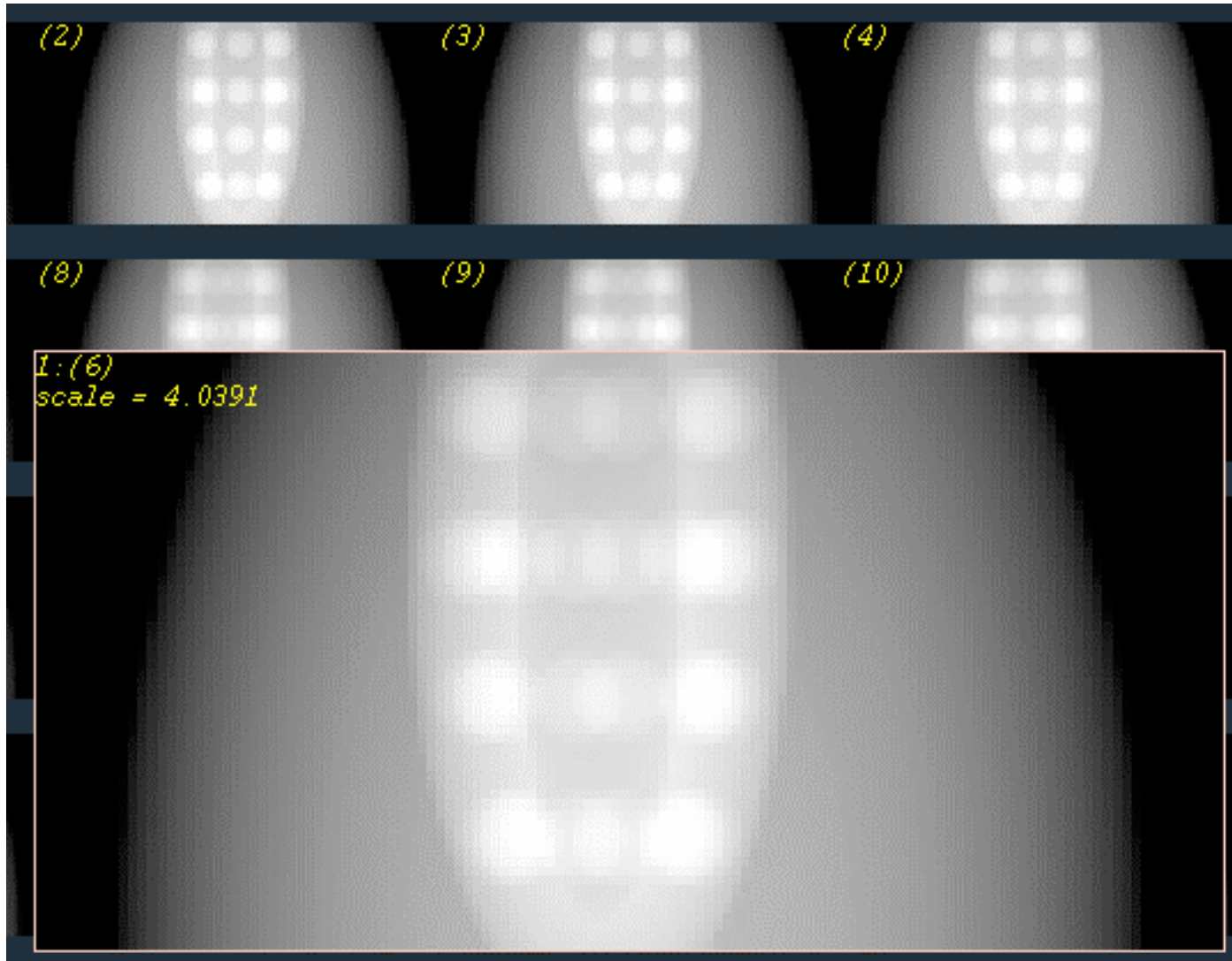
- Astronomia: Bracewell, 1956
- Medicina (revolução após Roentgen, 1895)
 - Primeiras publicações: Oldendorf, 1961
 - Primeiros experimentos: Kuhl (UPENN, 1963)
 - Equipamento médico: G Hounsfield (EMI, UK, 1971) e A Cormack (Tufts Univ) => Nobel, 1979

Estudo de caso: Phantom 3D

- background
- myocard.
- spots

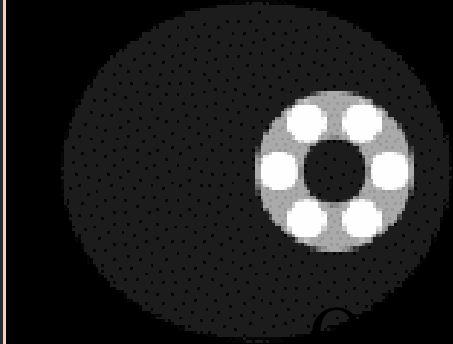


Dados disponíveis: Projeções



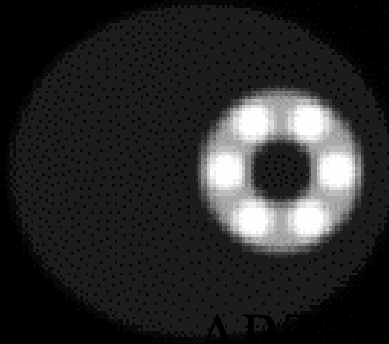
Processamento: Reconstrução

1:(6)
scale=1.3516



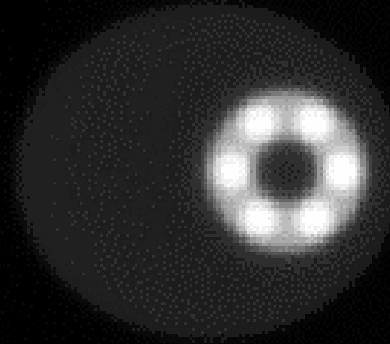
icon

2:(6)
scale=1.3516



icon

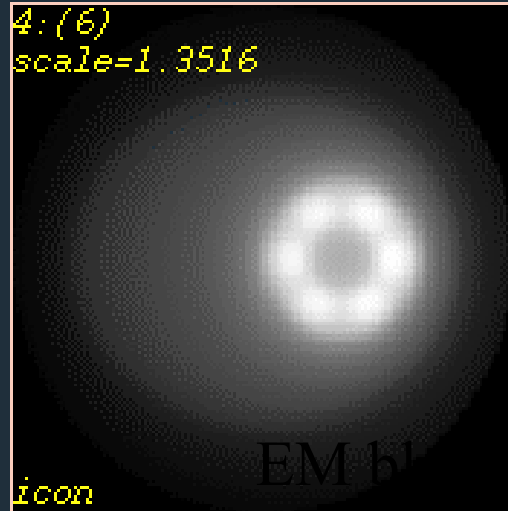
3:(6)
scale=1.3516



icon

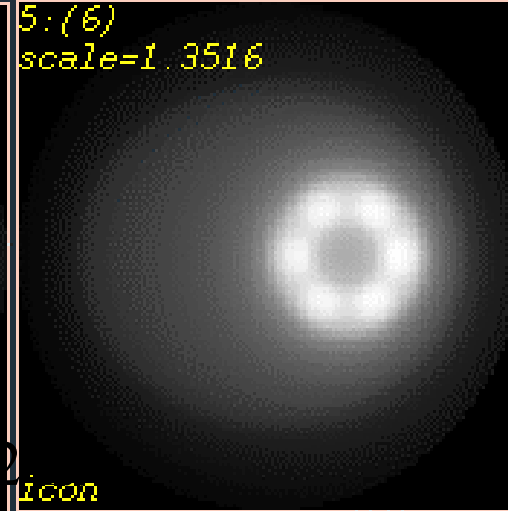
ART vox 2 it.

4:(6)
scale=1.3516



icon

5:(6)
scale=1.3516

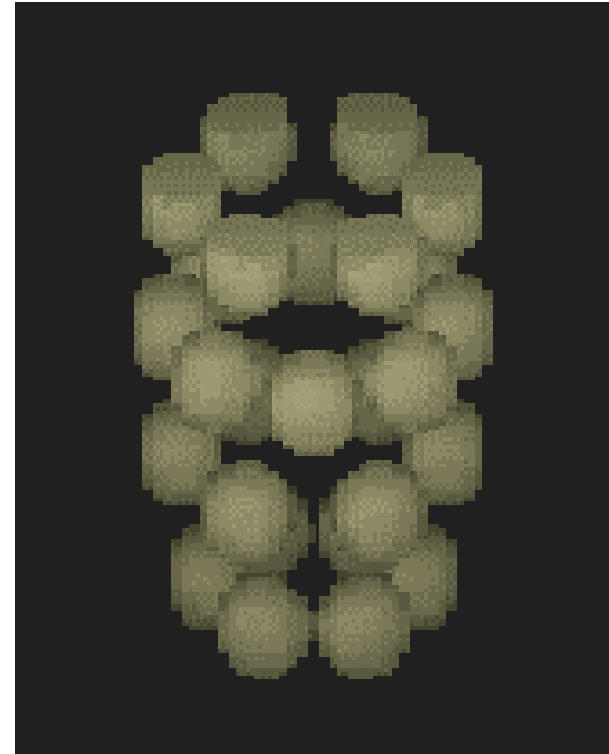


icon

EM vox 2 it.

- Noiseless proj.

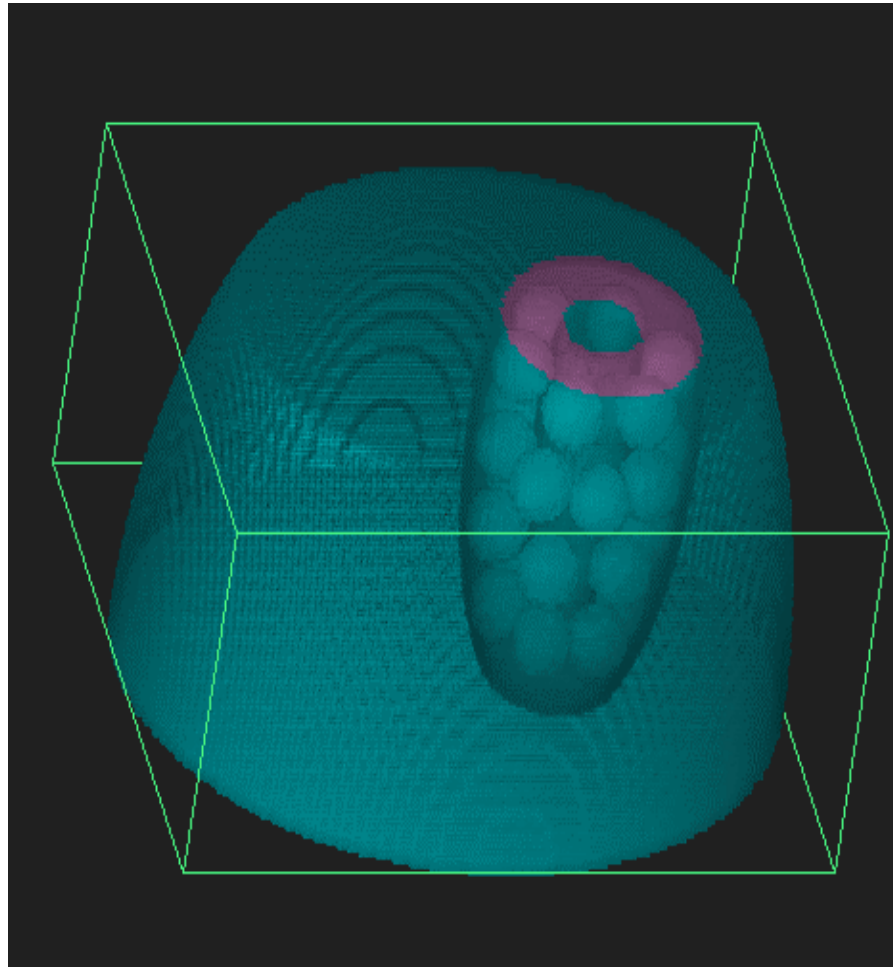
Motivação: 3D Reconstruction



- ART Blob
- Noisyless data
- 2 iterations

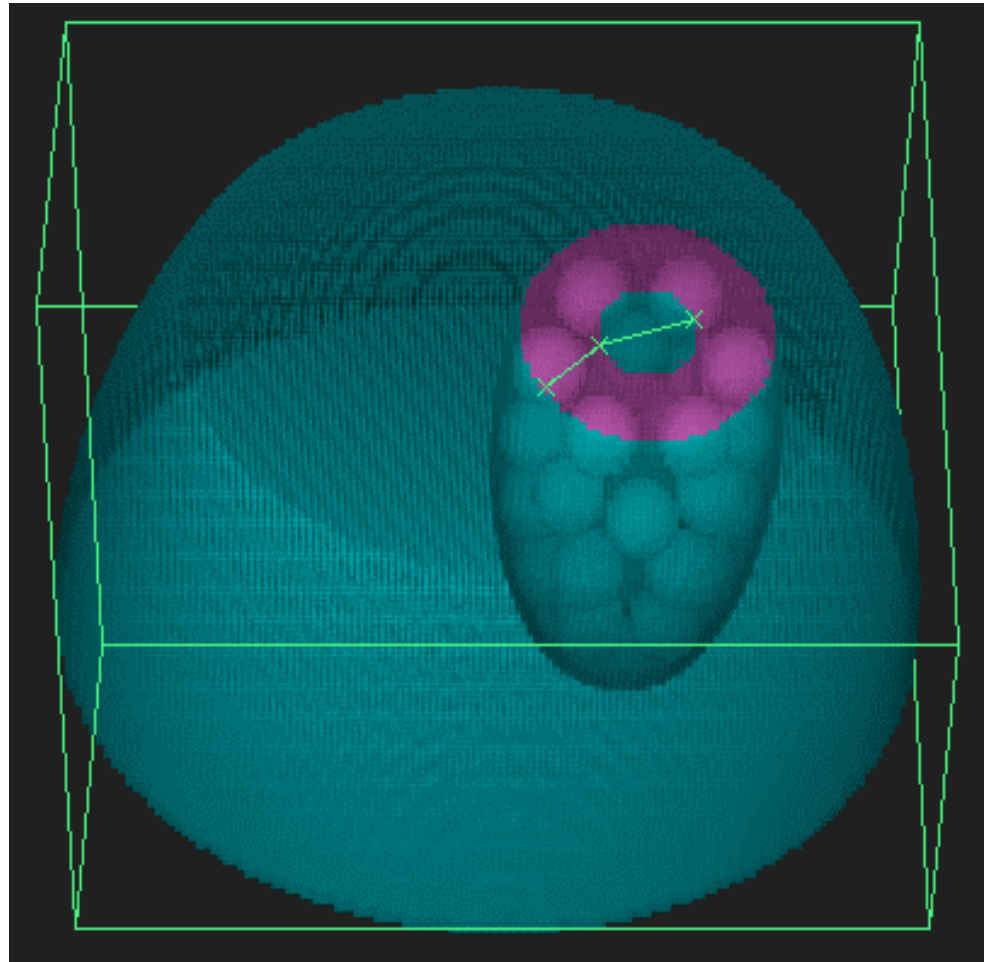
Motivação: 3D rendering (surface)

- phantom
- segmentation
- surface rendering



Motivation: Measuring in 3D

- distance
- area
- volume
- ejection fraction
- velocity
- ...



❑ Projection data formation

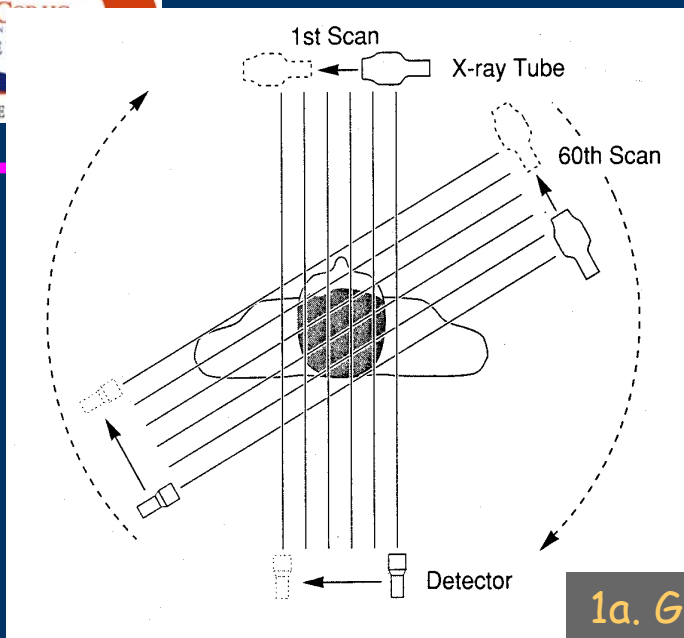
- CT, spiral CT, multi-slice spiral CT (0.5 mm)³, .5 s
- SPECT
- 3D PET

❑ Tomographic reconstruction methods

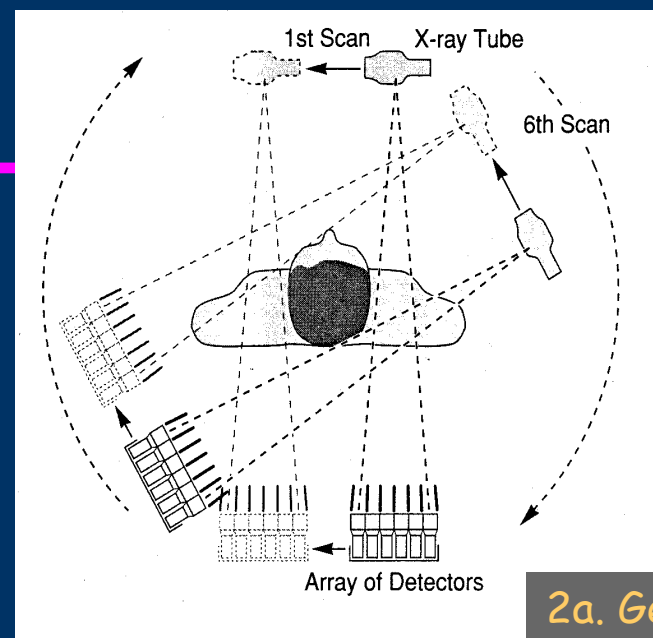
- ML-EM : Maximum-likelihood
- ART : Algebraic Reconstruction Technique
- FBP : Filtered Backprojection
- DFM : Direct Fourier Method

Gerações de tomógrafos por proj.

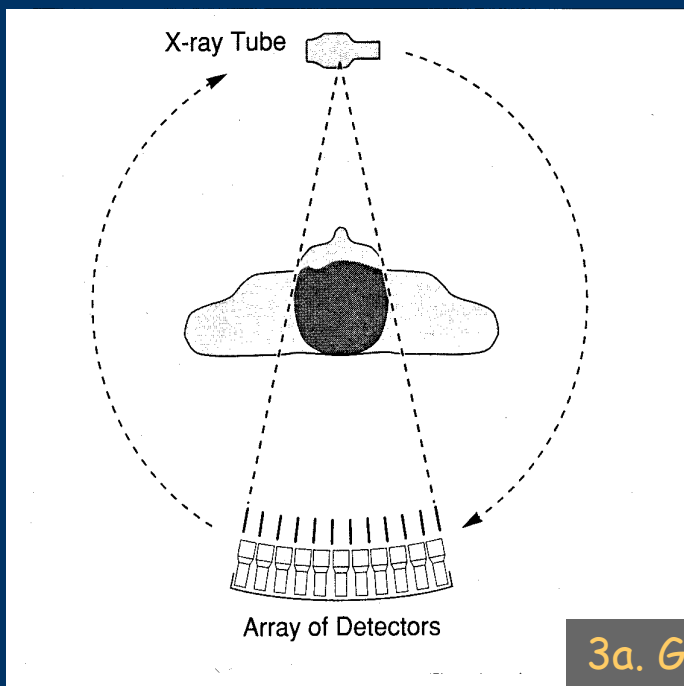
- ❑ Varredura de fonte unica e rotação da fonte-detetor
- ❑ Varredura de fonte conica e rotação
- ❑ Cone beam e rotação da fonte-detetor
 - spiral CT
 - multi-slice spiral CT
- ❑ Múltiplas fontes cônicas e detetores
- ❑ Electronic beam



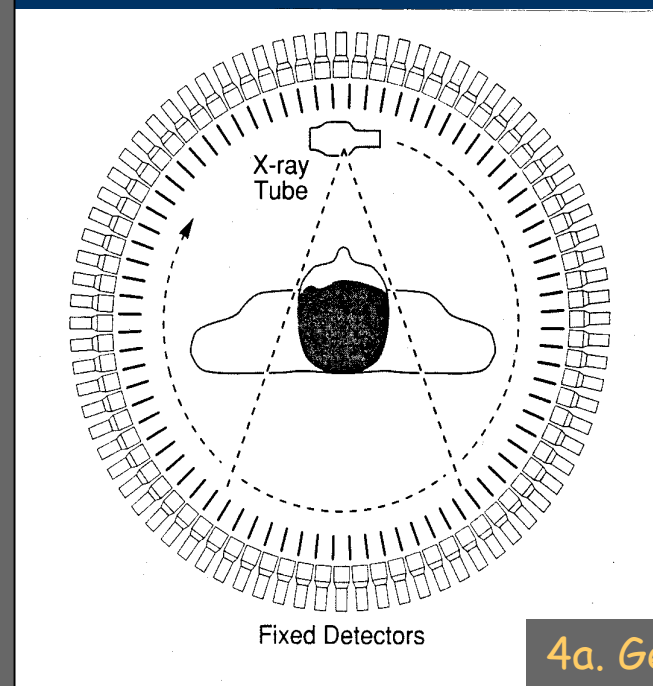
1a. Geração



2a. Geração

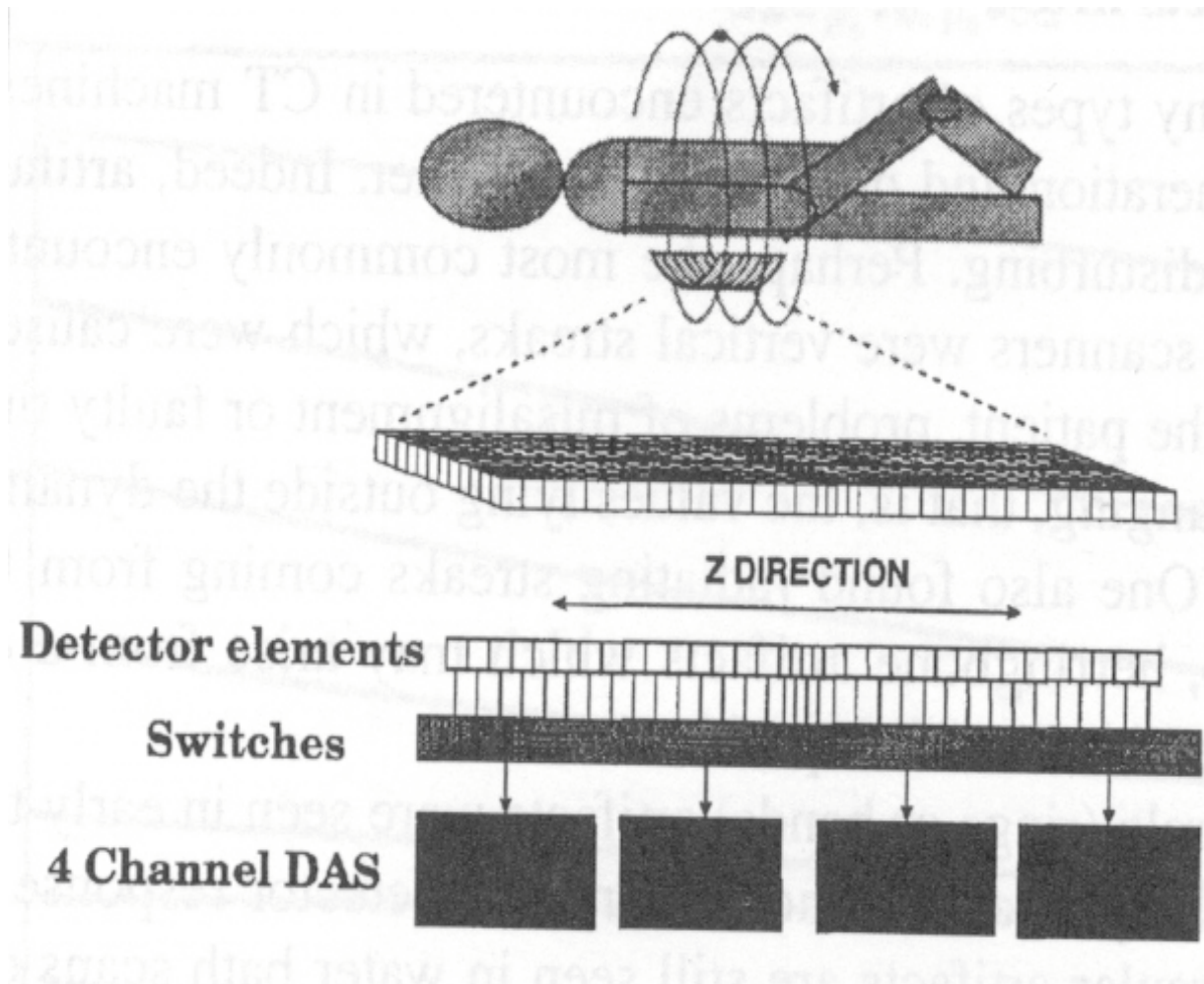


3a. Geração



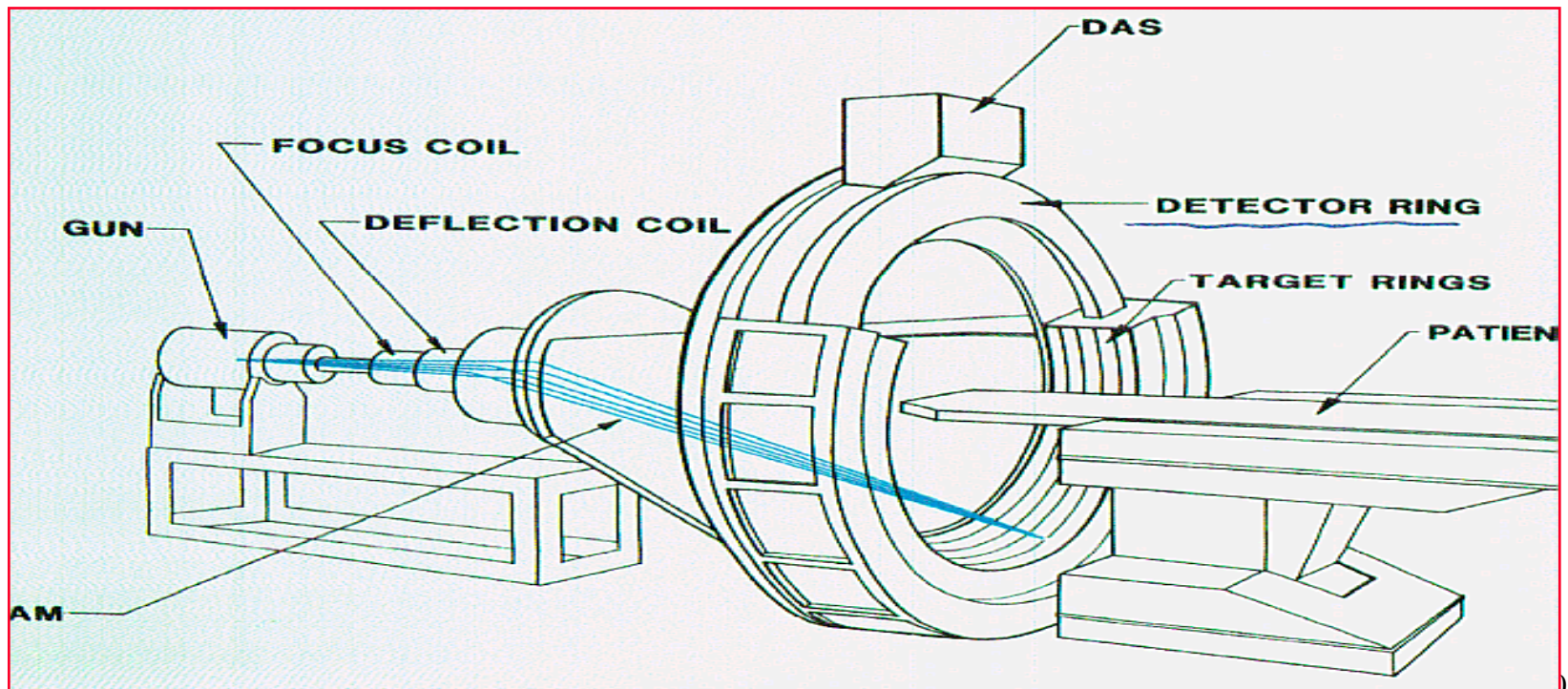
4a. Geração

Multi-slice Spiral (helical) CT

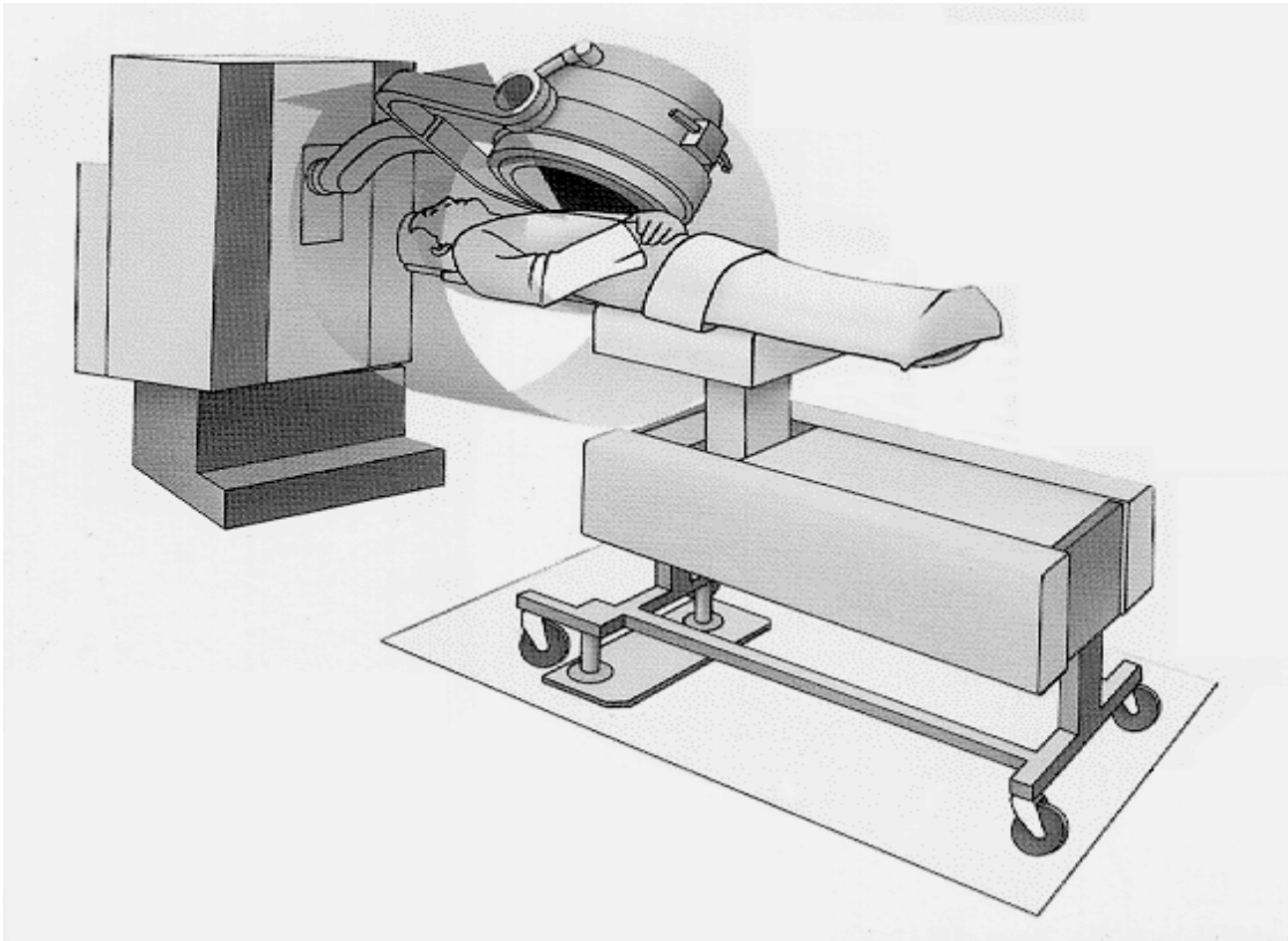


❑ Ultrafast CT

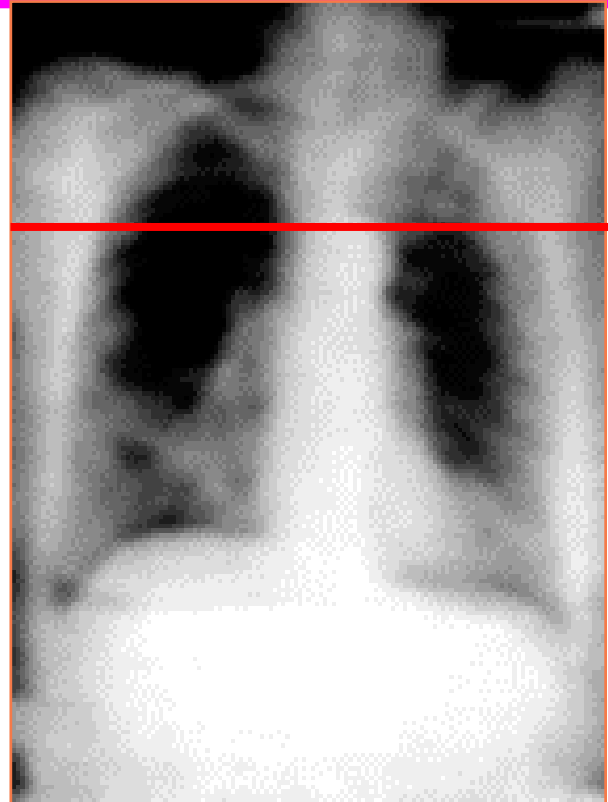
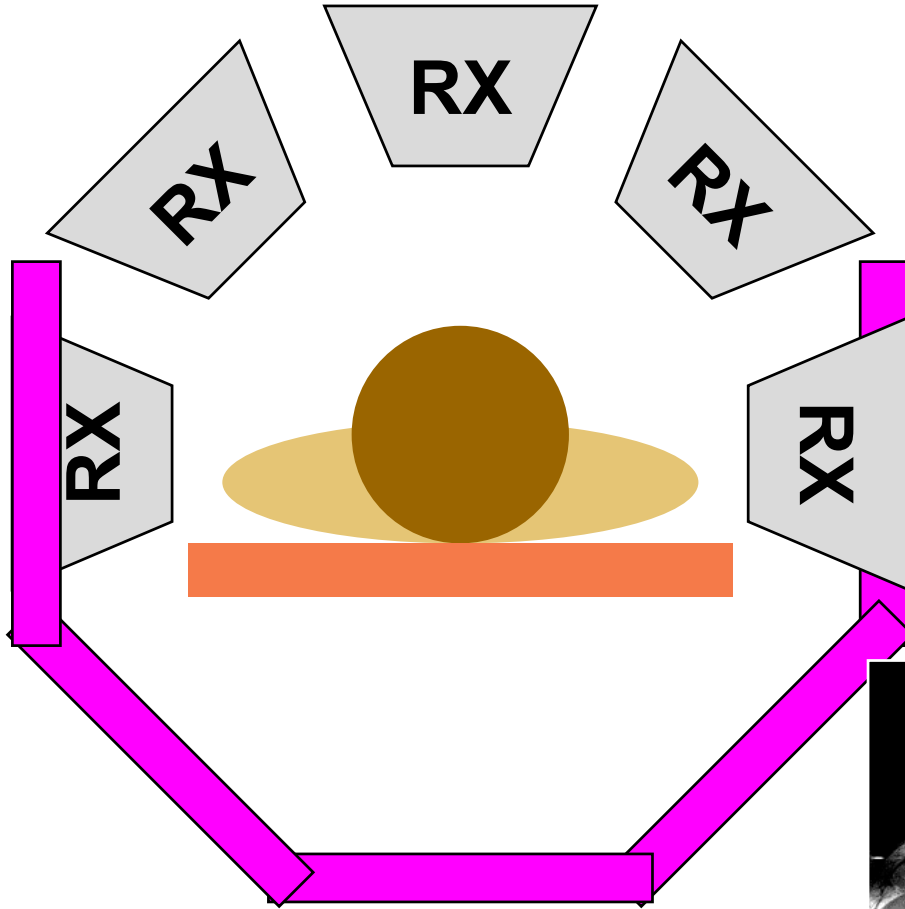
- sem estruturas móveis
- 50 ms/scan (20 cortes/s)
- volume: 8 cm em 0.25 s



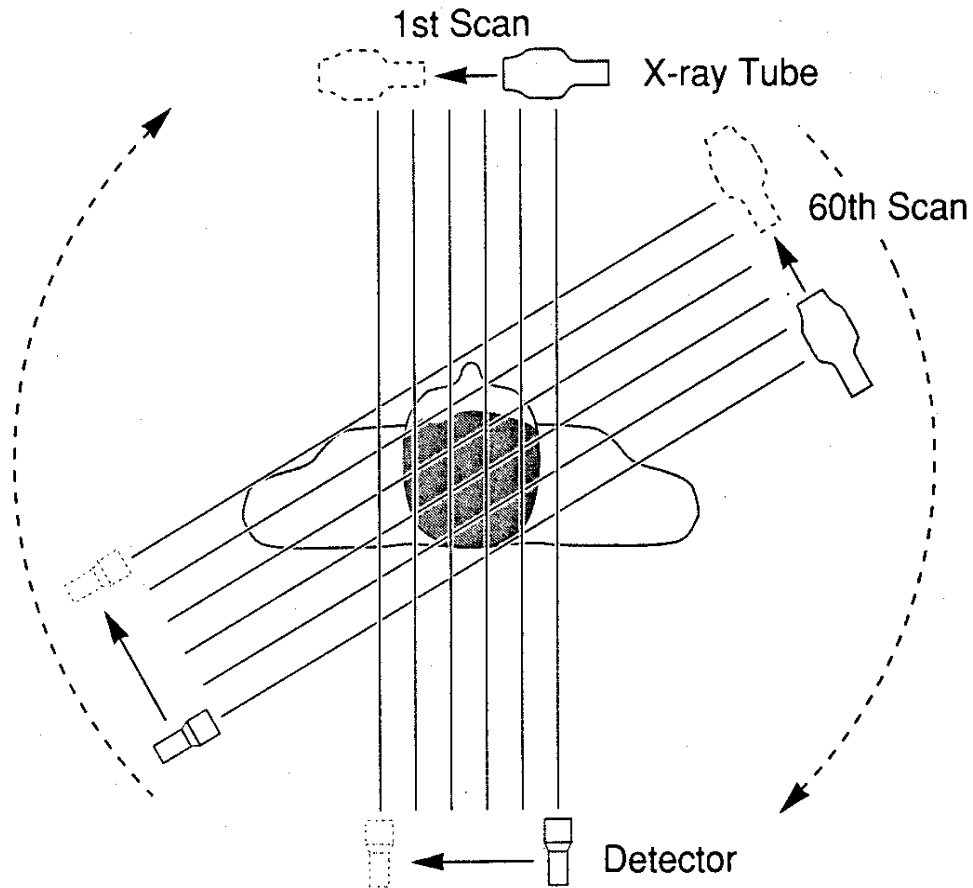
SPECT: Single Photon Emission CT



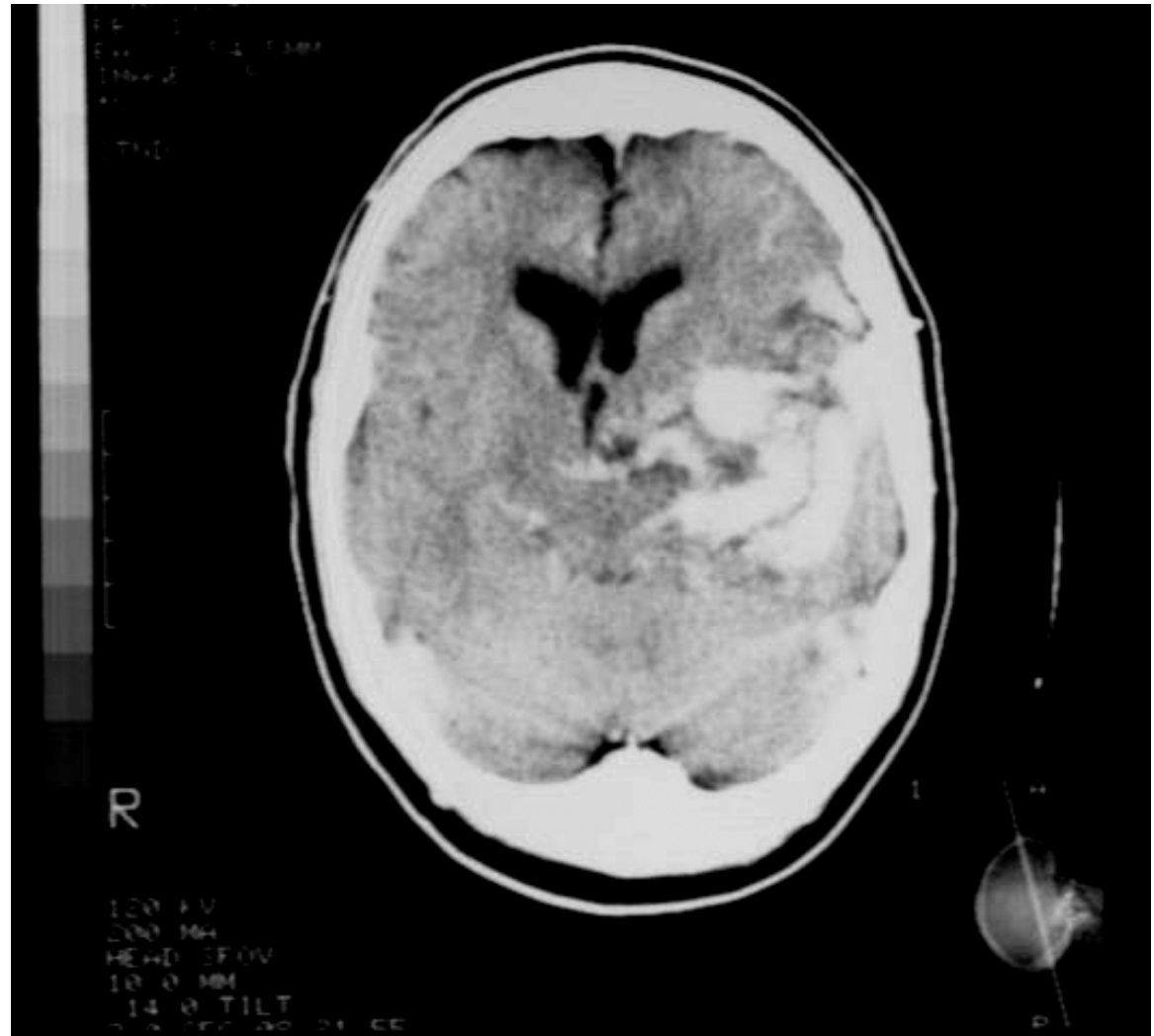
CT (raio-X): o que há de comum?



Linhas de projeção

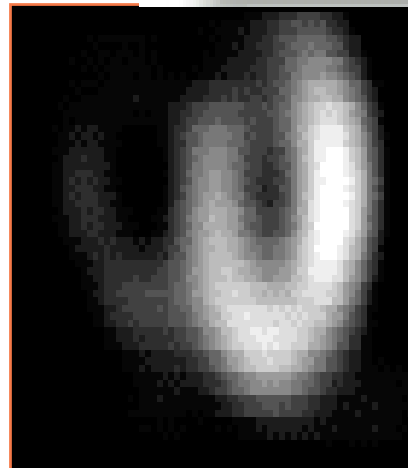
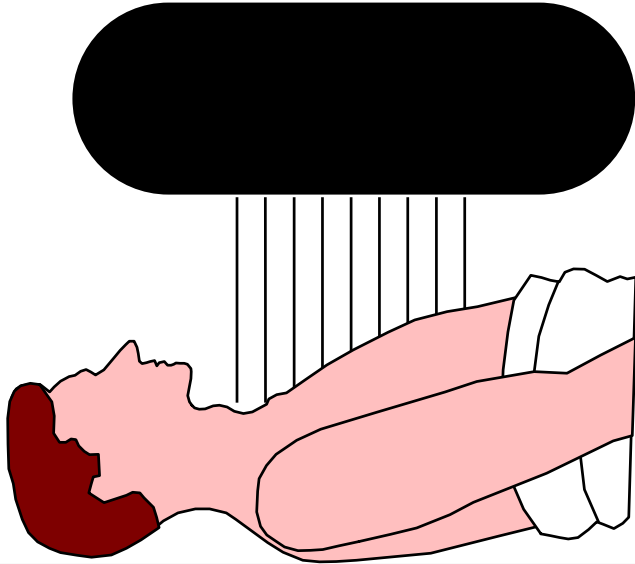


CT



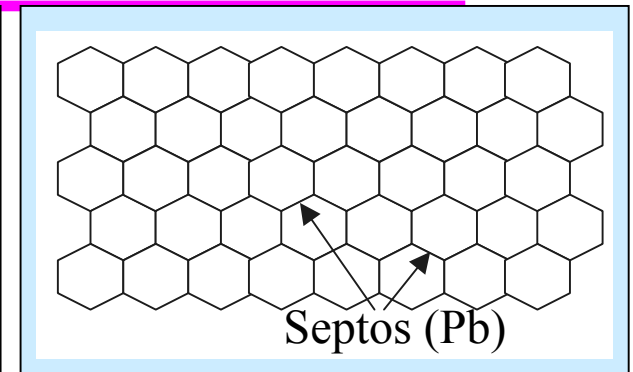
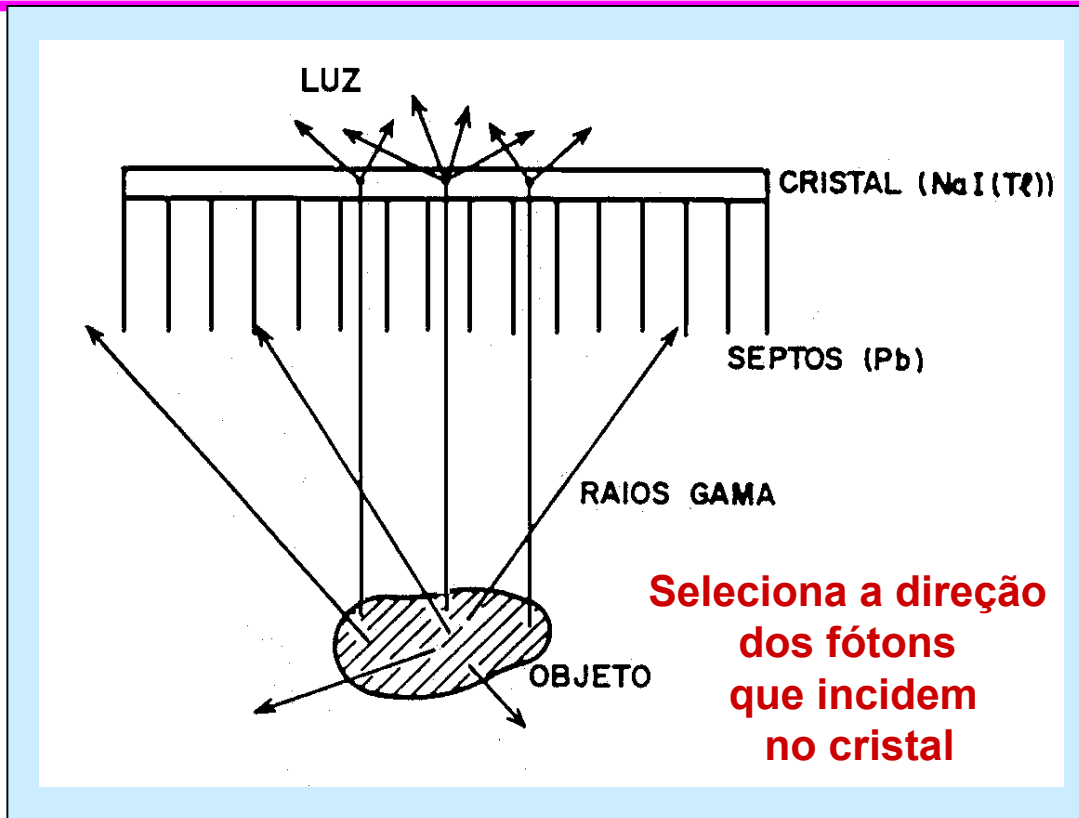
MN: SPECT - o que há de comum?

❑ Medicina Nuclear:SPECT



**Miocárdio
marcado com
material radioativo
(Tc)**

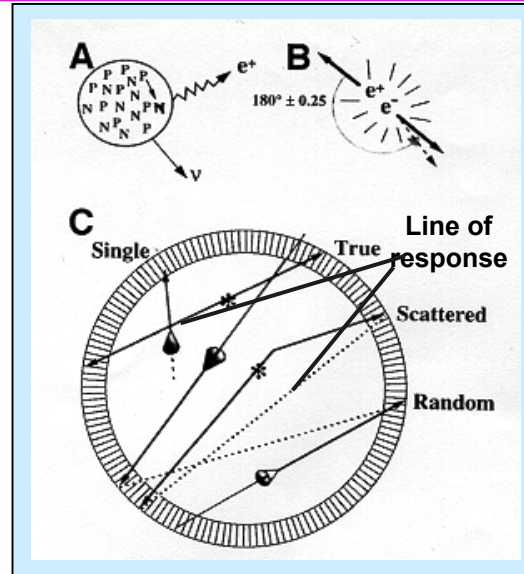
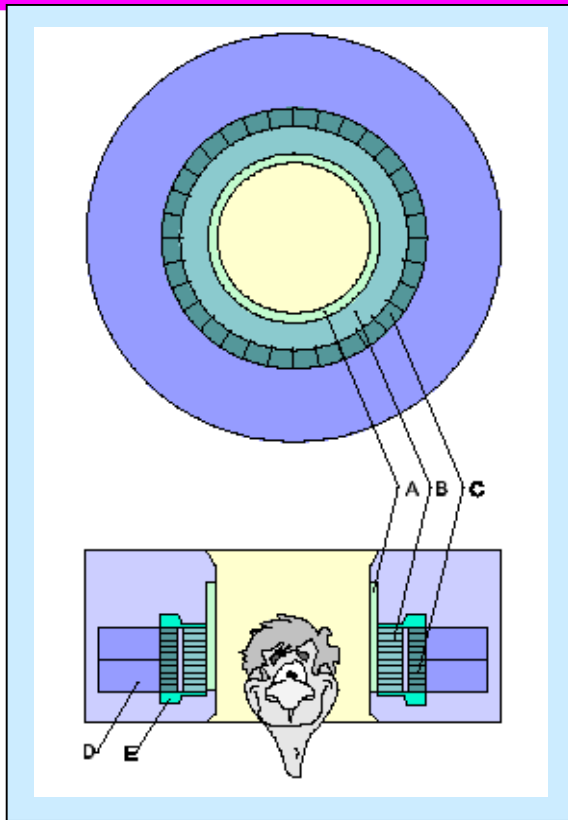
SPECT: Colimador (septos paralelos)



vista superior

A imagem formada no cristal é uma projeção bidimensional da distribuição tridimensional do radiofármaco no organismo

MN: PET - o que há de comum?

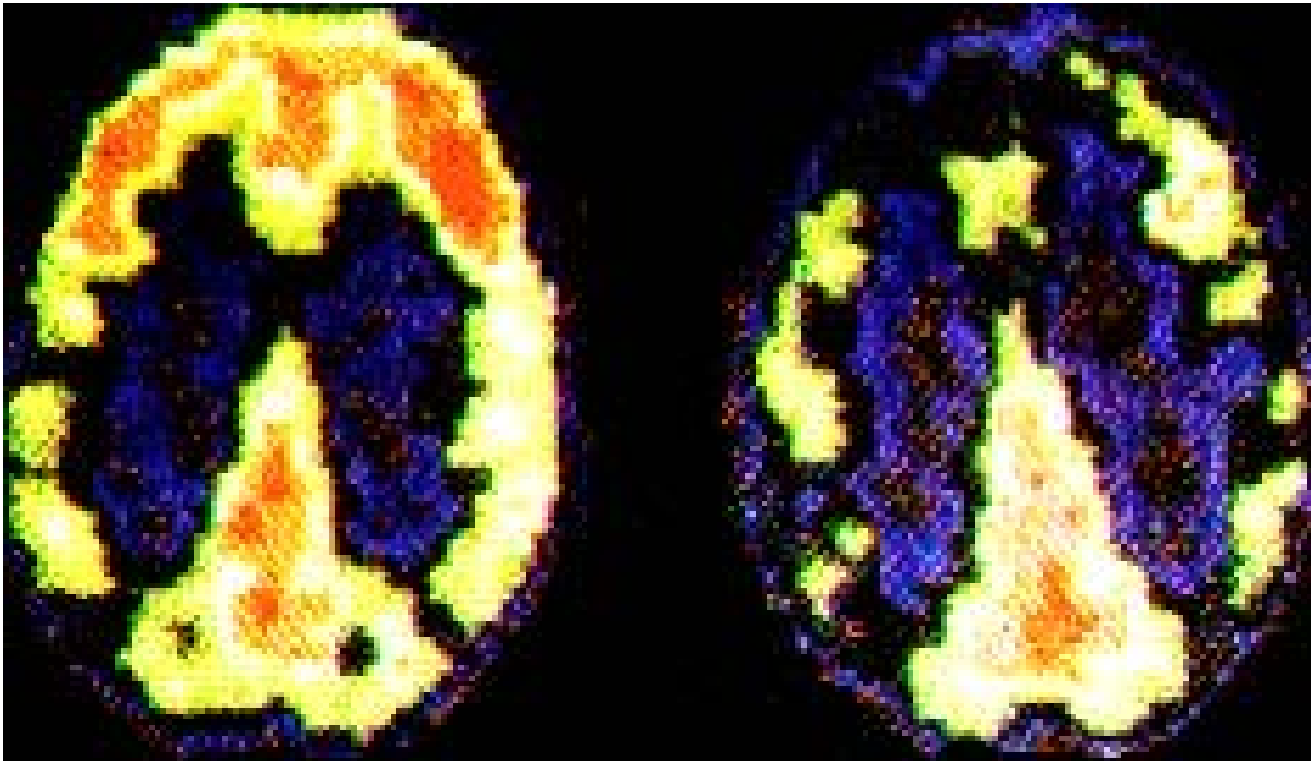


$\Delta t_{\text{coinc}} = 10 \sim 12 \text{ ns}$

- A.** fontes para mapas de atenuação
- B.** absorvedores de fótons espalhados
- C.** blocos de detetores (BGO)
- D.** fotomultiplicadoras
- E.** blindagem



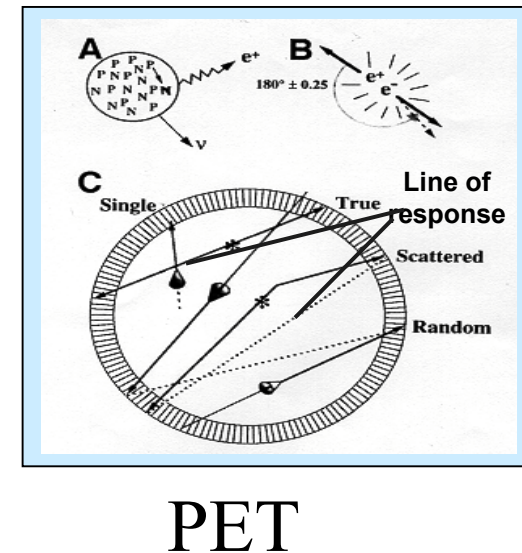
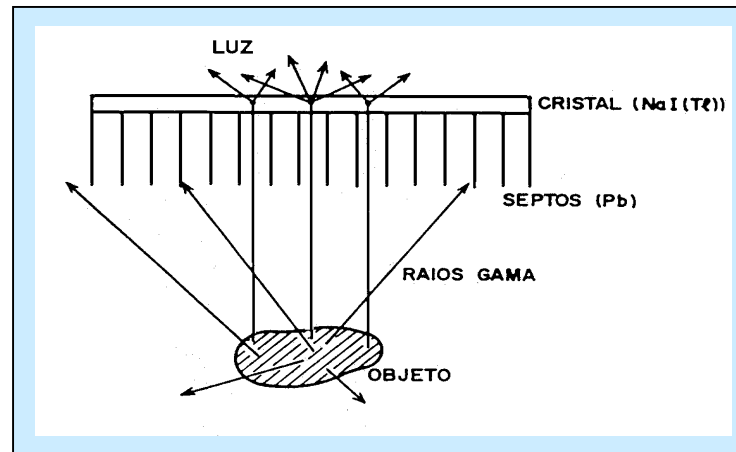
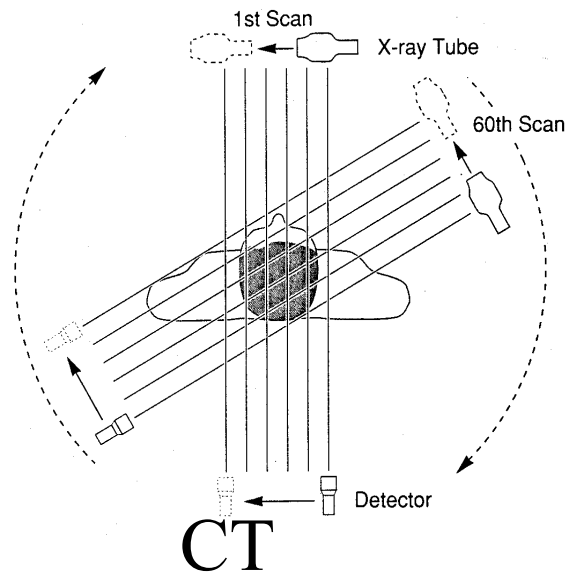
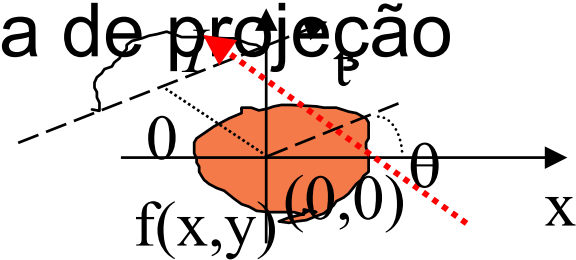
❑ Reconstrução Tomográfica (PET Cerebral)



O que há de comum:

Tomografia a partir das projeções

- cada pixel na projeção contém informação acumulada ao longo da linha de projeção



O que há de diferente?

- ❑ Entre CT, SPECT e PET?

Tomografia por Transmissão (CT)

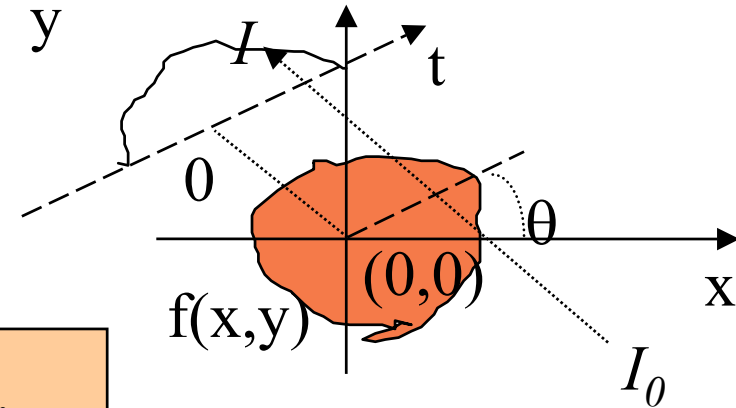
$$I = I_0 \cdot \exp\left(-\int_L f(x, y) \cdot ds\right)$$

$$\ln\left(\frac{I_0}{I}\right) = \int_L f(x, y) \cdot ds \quad (\text{Integral de linha})$$

$$p_i = \sum_j \mu_j \cdot d_{ij}$$

Desconhecido 1 conjunto:

→ $\mu_j \quad j = 1, N$



Transformada de Radon 2D
(projection operator)

$f(x, y)$

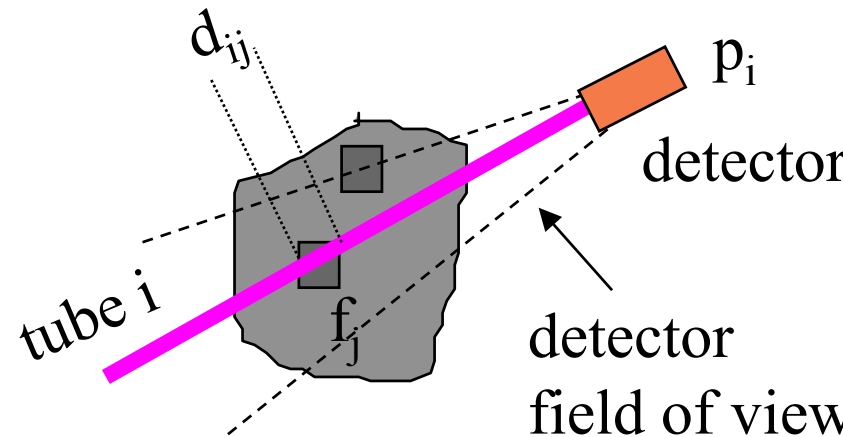
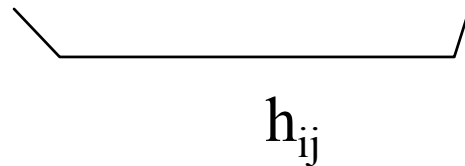


$g(t, \theta)$

(sinograma)

SPECT

$$p_i = \sum_j f_j \cdot d_{ij} \cdot A \cdot \exp\left(- \int_{s_{ij} \rightarrow \infty} \mu_j(s) ds\right)$$



Desconhecidos 2 conjuntos:

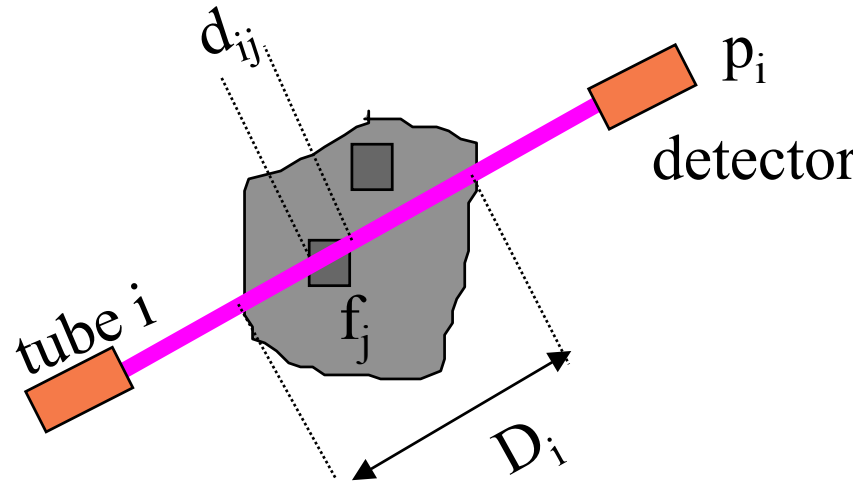
⇒ $f_j \quad j=1, N$

μ_j

PET

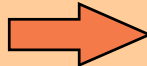
$$p_i = \sum_j f_j \cdot (A \cdot d_{ij}) \cdot \exp\left(-\int_{s_j} \mu_j(s) ds\right)$$

$$p_i = \exp\left(-\int_{D_i} \mu(s) ds\right) \cdot \sum_j f_j \cdot (A \cdot d_{ij})$$



- accumulated attenuation estimation: simpler
 - Fonte externa => custo, tempo maior

$$\frac{p_i}{\exp\left(-\int_{D_i} \mu(s) ds\right)} = \sum_j f_j \cdot (A \cdot d_{ij})$$

Desconhecido 1 conjunto:
 $f_j \quad j = 1, N$

O que há de diferente?

❑ Entre CT, SPECT e PET?

- CT: sist. Eq. Lineares, e 1 conjunto de incógnitas
- SPECT: sist. Eq. Não-lineares, e 2 conjuntos de incóg.
- PET: similar a CT se houver boa estimativa das aten.

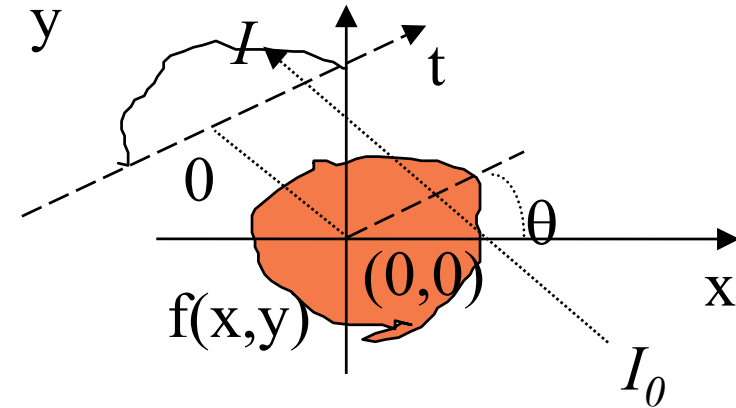
Ou similar a SPECT => algoritmos AA

Tomografia por Transmissão (CT)

$$I = I_0 \cdot \exp\left(-\int_L f(x, y) \cdot ds\right)$$

$$\ln\left(\frac{I_0}{I}\right) = \int_L f(x, y) \cdot ds \quad (\text{Integral de linha})$$

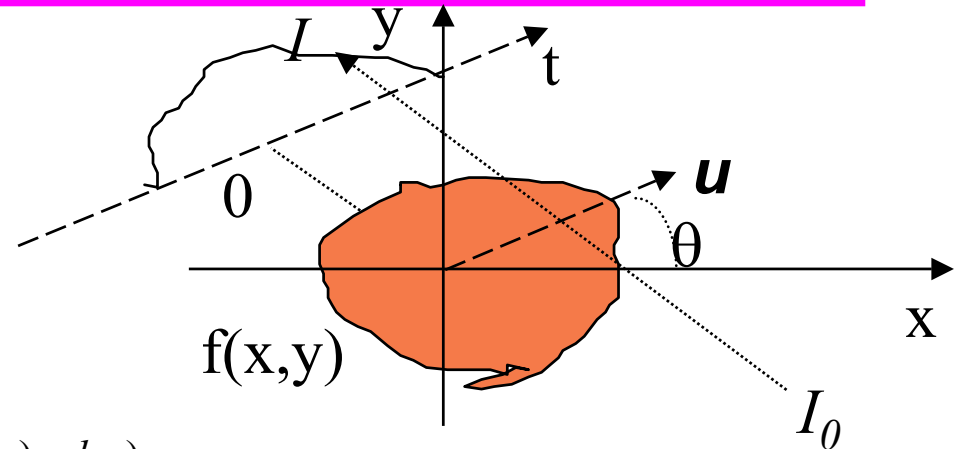
$$\begin{aligned} g(t, \theta) &= R f = \int_L f(x, y) \cdot ds \\ &= \iint f(x, y) \cdot \delta(x \cdot \cos \theta + y \cdot \sin \theta - t) \cdot dx \cdot dy \end{aligned}$$



Transformada de Radon 2D
(projection operator)

$f(x, y)$ \longleftrightarrow $g(t, \theta)$ (sinograma)

Transf. Radon 2D



$$I = I_0 \cdot \exp\left(-\int_L f(x, y) \cdot ds\right)$$

$$\ln\left(\frac{I_0}{I}\right) = \int_L f(x, y) \cdot ds \quad (\text{Integral de linha})$$

$$g(t, \theta) = \mathbf{R} f = \int_L f(x, y) \cdot ds$$

$$= \iint f(x, y) \cdot \delta(x \cdot \cos \theta + y \cdot \sin \theta - t) \cdot dx \cdot dy$$

Transformada de Radon 2D
(projection operator)

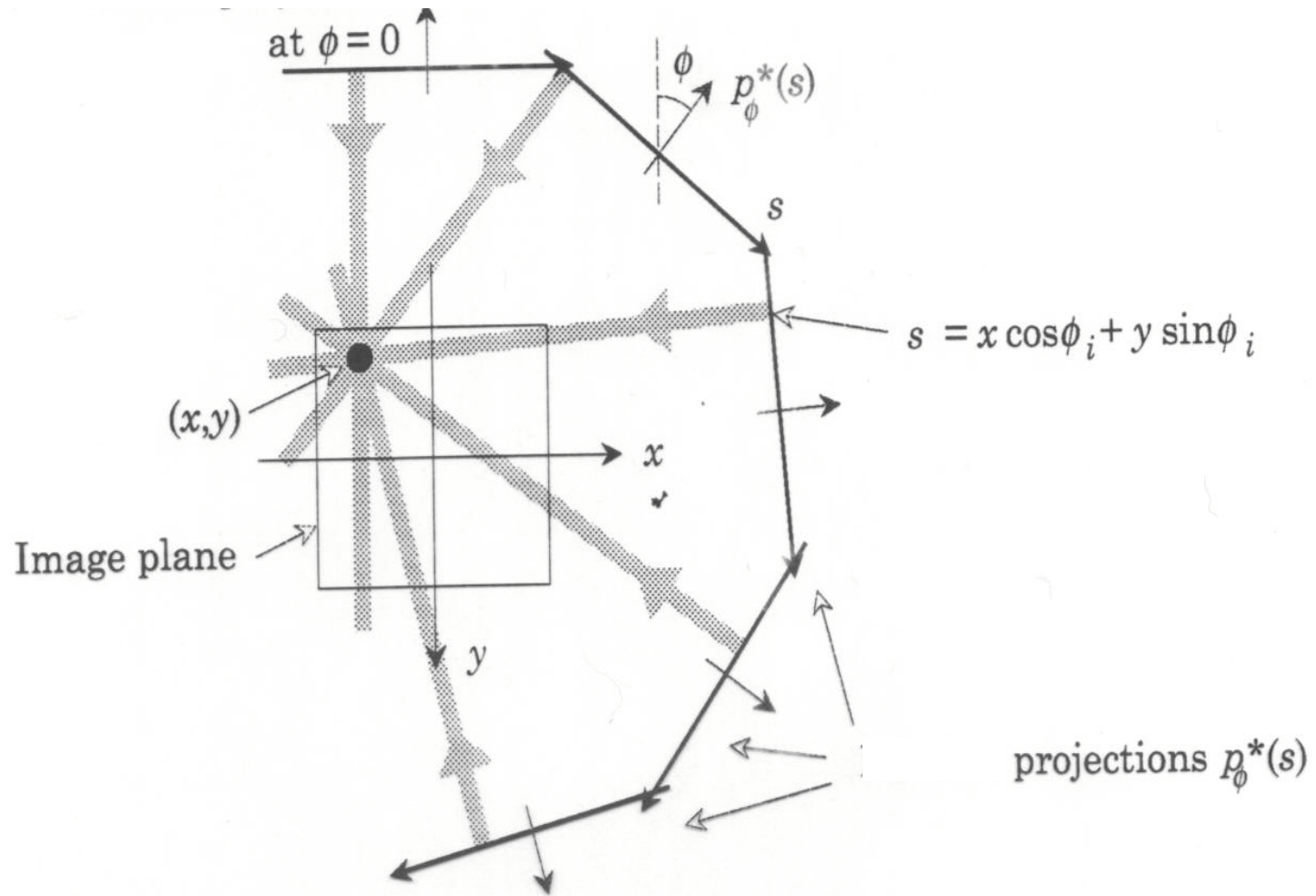
$f(x, y)$



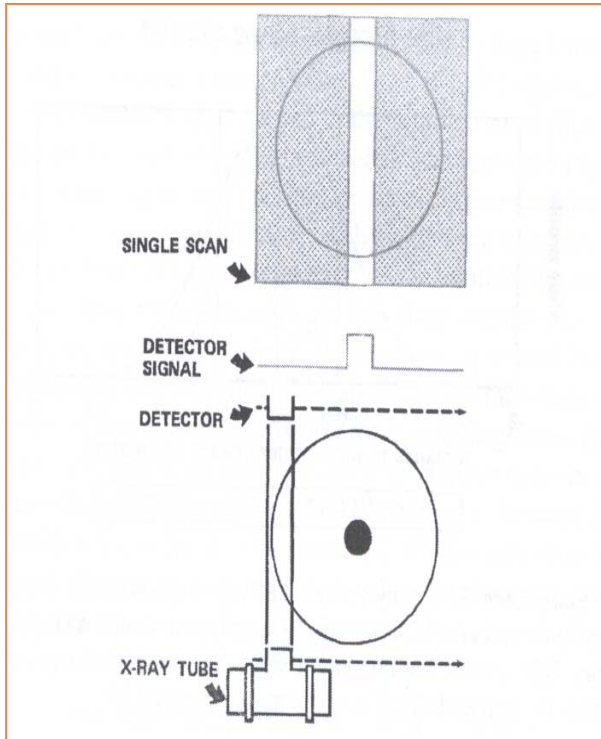
$g(t, \theta)$

(sinograma)

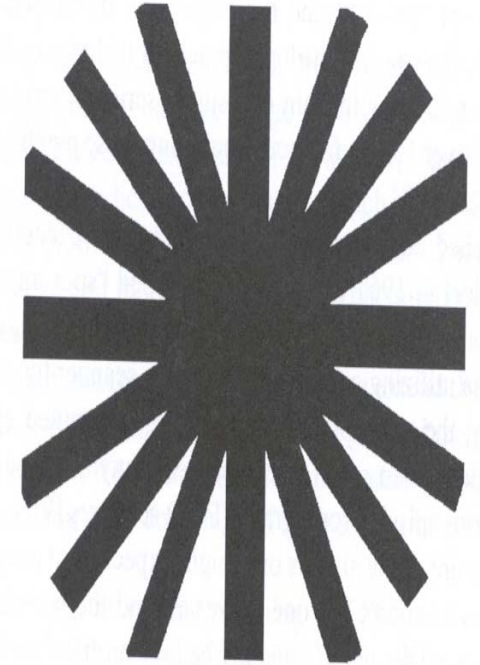
Solução?



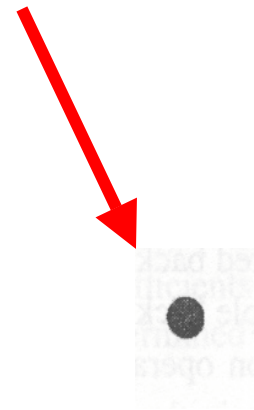
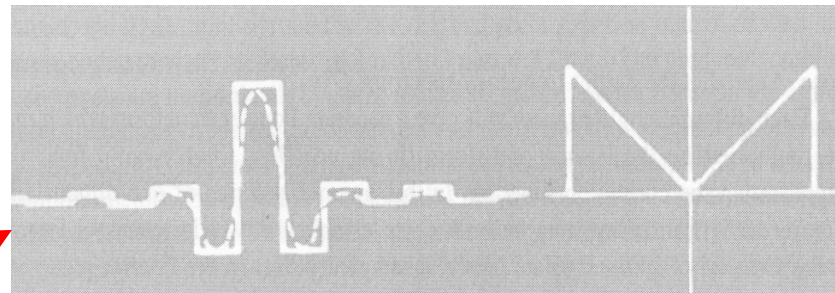
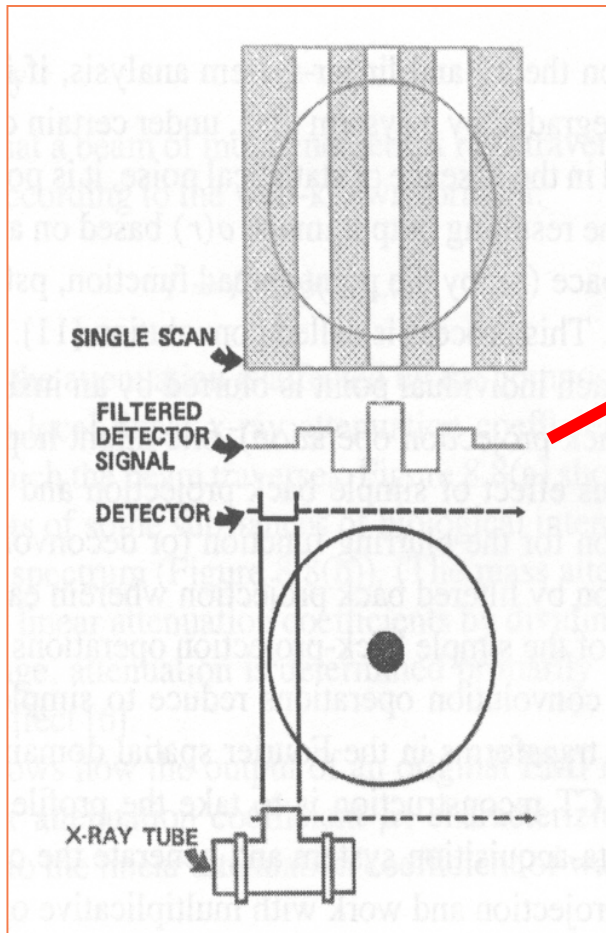
Reconstrução ingênua (backprojection)



Reprojeção
simples



Reconstrução com filtered backprojection



Algébrica

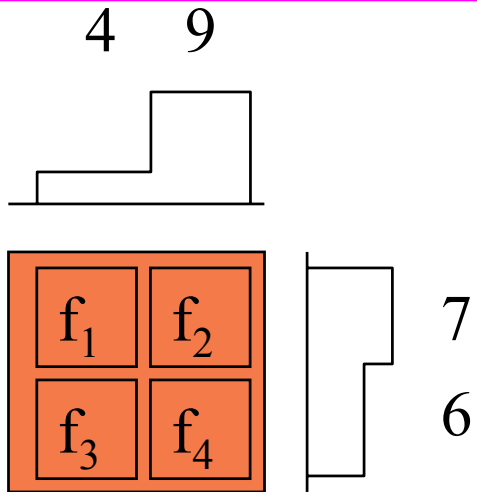


Imagem
f

Problema: $f \mid$

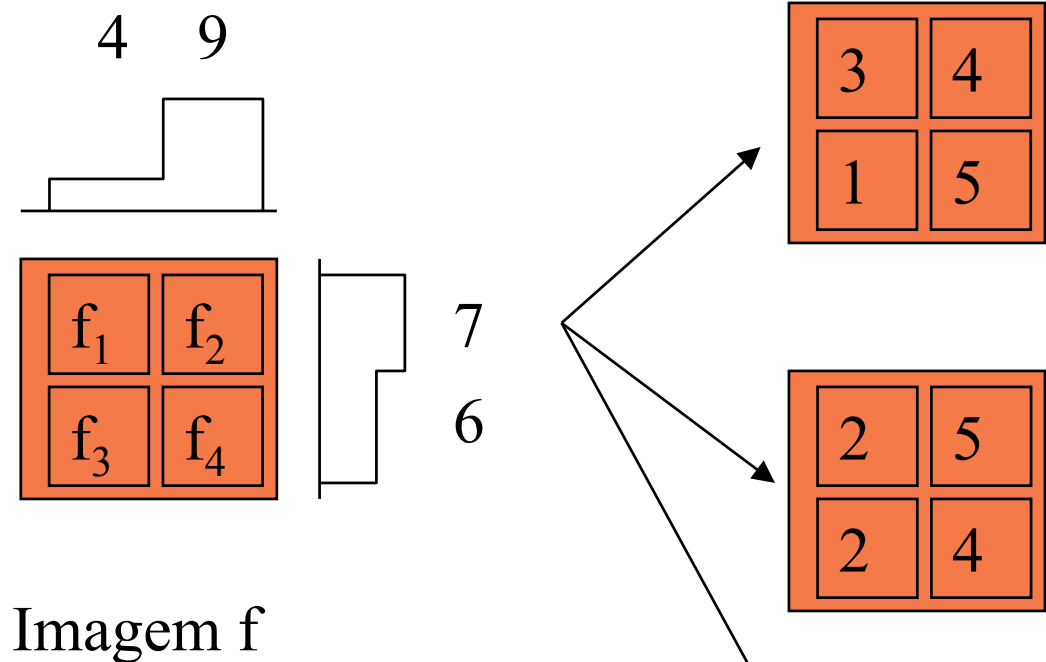
$$f_1 + f_2 = 7$$

$$f_3 + f_4 = 6$$

$$f_1 + f_3 = 4$$

$$f_2 + f_4 = 9$$

Soluções



$$A \cdot x = b$$

M equações com N incógnitas

Sistema indeterminado (infinitas soluções, $\text{rank} < N$)

Sistema inconsistente ($M \text{ eq. Lin. Indep} > N$) \Rightarrow otimização

Algébrica: otimização (regularizada)

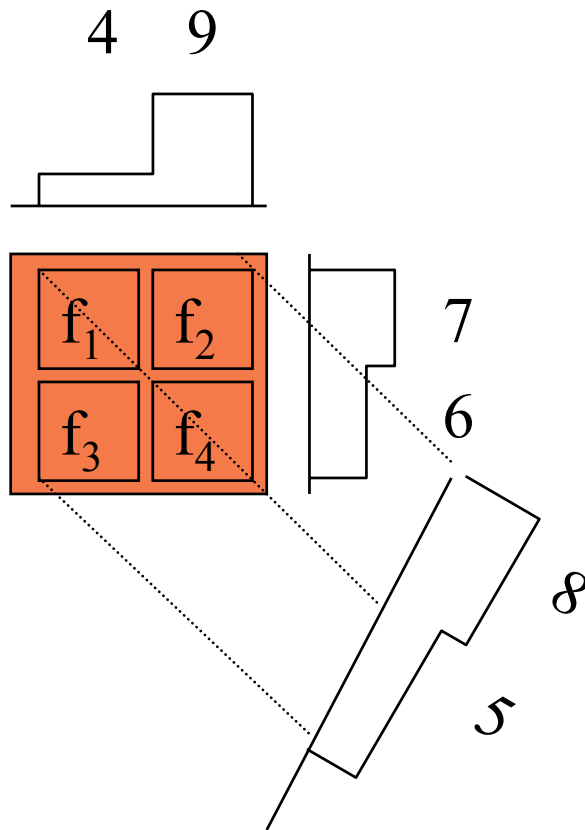


Imagem f

Problema: $f \mid$

$$f_1 + f_2 = 7$$

$$f_3 + f_4 = 6$$

$$f_1 + f_3 = 4$$

$$f_2 + f_4 = 9$$

$$.5f_1 + f_3 + .5f_4 = 5$$

$$.5f_1 + f_2 + .5f_4 = 8$$

Soluções (otimizada)

$$A \cdot x = b$$

6 equações com 4 incógnitas

Sistema inconsistente (M eq. Lin. Indep > N) => otimização

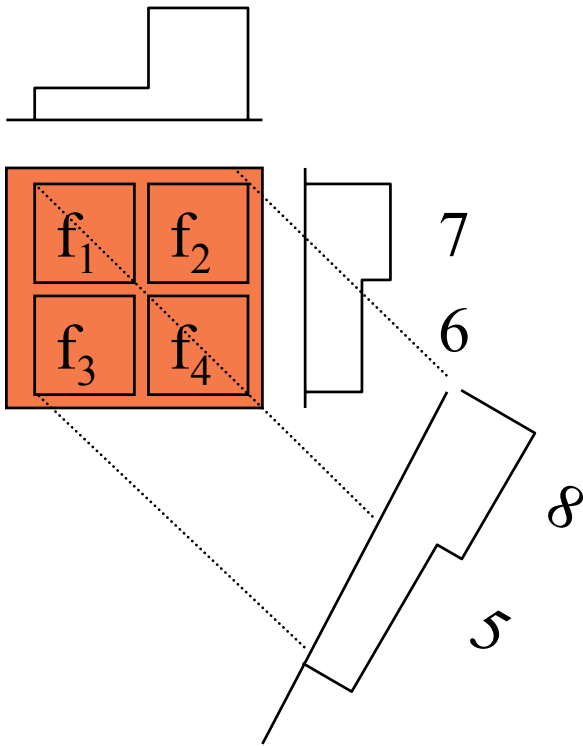
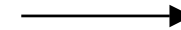


Imagem f

$$\min_{\hat{x}} | A \cdot \hat{x} - b |^2$$

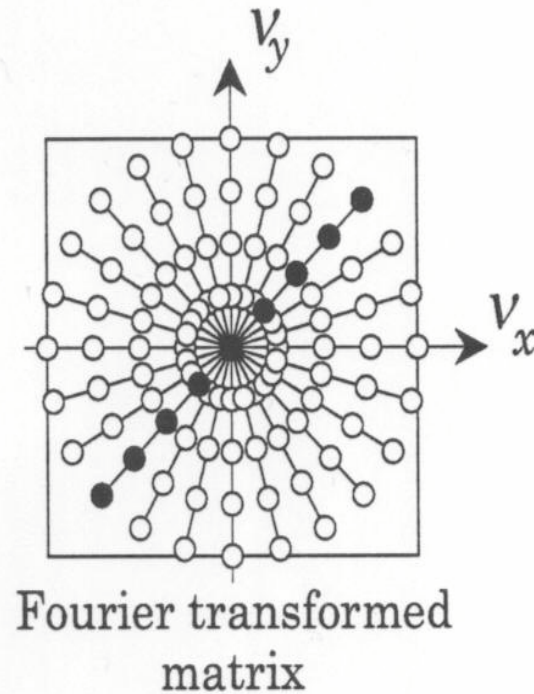
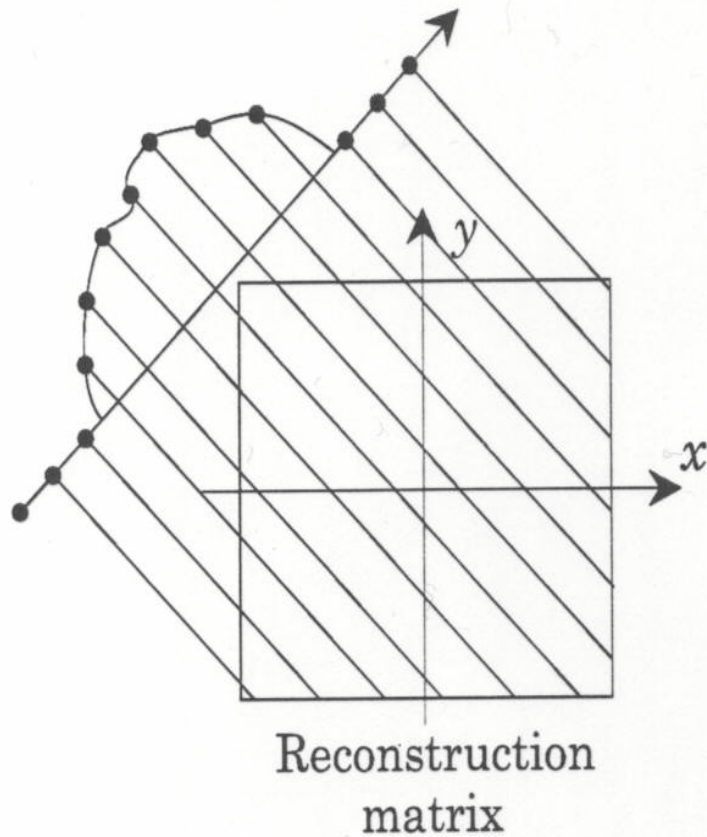
$$\hat{x} = A^+ b$$

$$A^+ = (A' A)^{-1} A'$$



2.5	4.6
2.0	4.1

Alguma outra solução ?



Teorema do corte central

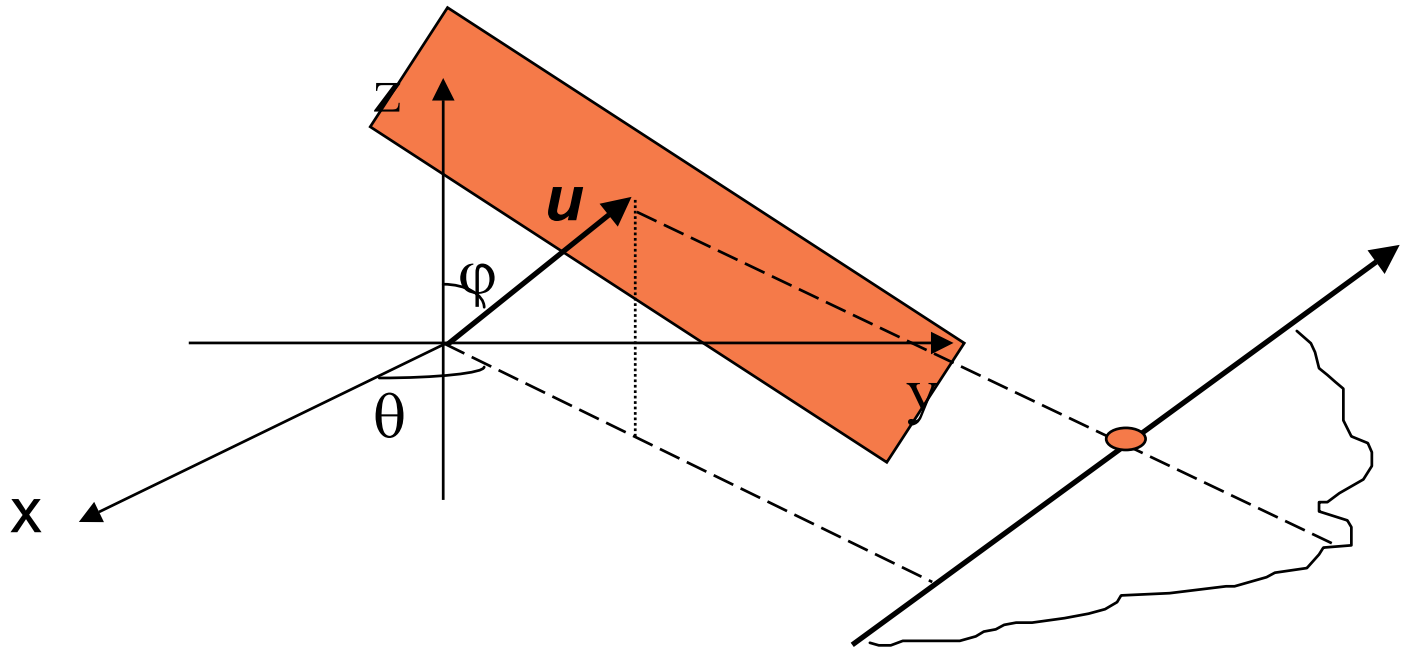
Transf. Radon 3D

$$g(t, \vec{u}) = R f = \int f(\vec{r}) \cdot \delta(\vec{r}^T \cdot \vec{u} - t) d\vec{r}$$

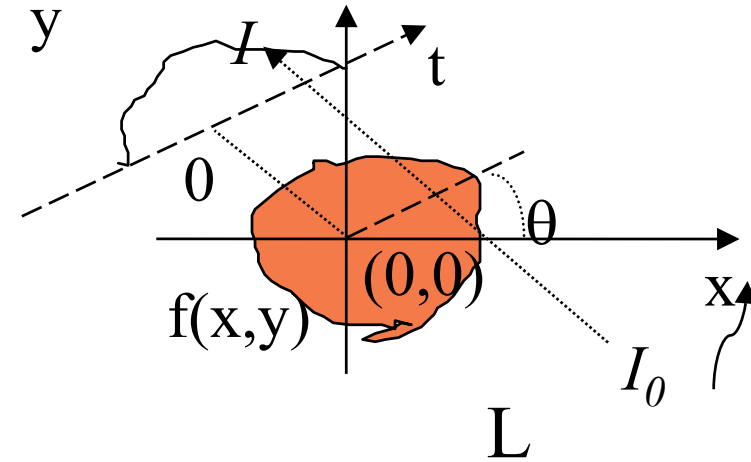
$$g(t, \vec{u}) = \iint f(\vec{r}) \cdot \delta(\vec{r}^T \cdot \vec{u} - t) d\vec{r}$$

$$g(t, \vec{u}) = \iiint f(x, y, z) \cdot$$

$$\delta(x \cdot \sin \varphi \cdot \cos \theta + y \sin \varphi \cdot \sin \theta + z \cdot \cos \varphi - t) dx \cdot dy \cdot dz$$



Tomografia por Transmissão (CT)



$$g(t, \theta) = R f = \int_L f(x, y).ds$$

$$= \iint f(x, y).\delta(x.\cos \theta + y.\sin \theta - t).dx.dy$$

Transformada de Radon 2D
(projection operator)

$f(x,y)$ \longleftrightarrow $g(t,\theta)$ (sinograma)

Teorema da Projeção

$$g(t, \theta) = \iint f(x, y) \cdot \delta(x \cdot \cos\theta + y \cdot \sin\theta - t) \cdot dx \cdot dy$$

$$G(u, \theta) \equiv \iint f(x, y) \int \delta(x \cdot \cos\theta + y \cdot \sin\theta - t) \cdot e^{-j2\pi ut} dt \cdot dx \cdot dy$$

$$G(u, \theta) = \iint f(x, y) \cdot e^{-j2\pi u(x \cdot \cos\theta + y \cdot \sin\theta)} dx \cdot dy$$

$$\therefore G(u, \theta) = F(u \cdot \cos\theta, u \cdot \sin\theta) \Rightarrow \text{DFM}$$

Transf. Radon Inversa

$$f(x, y) = \iint F(u, v) \cdot e^{j2\pi \cdot (x \cdot u + y \cdot v)} du \cdot dv$$

coord. polares $\Rightarrow u = w \cdot \cos\theta$ e $v = w \cdot \sin\theta$

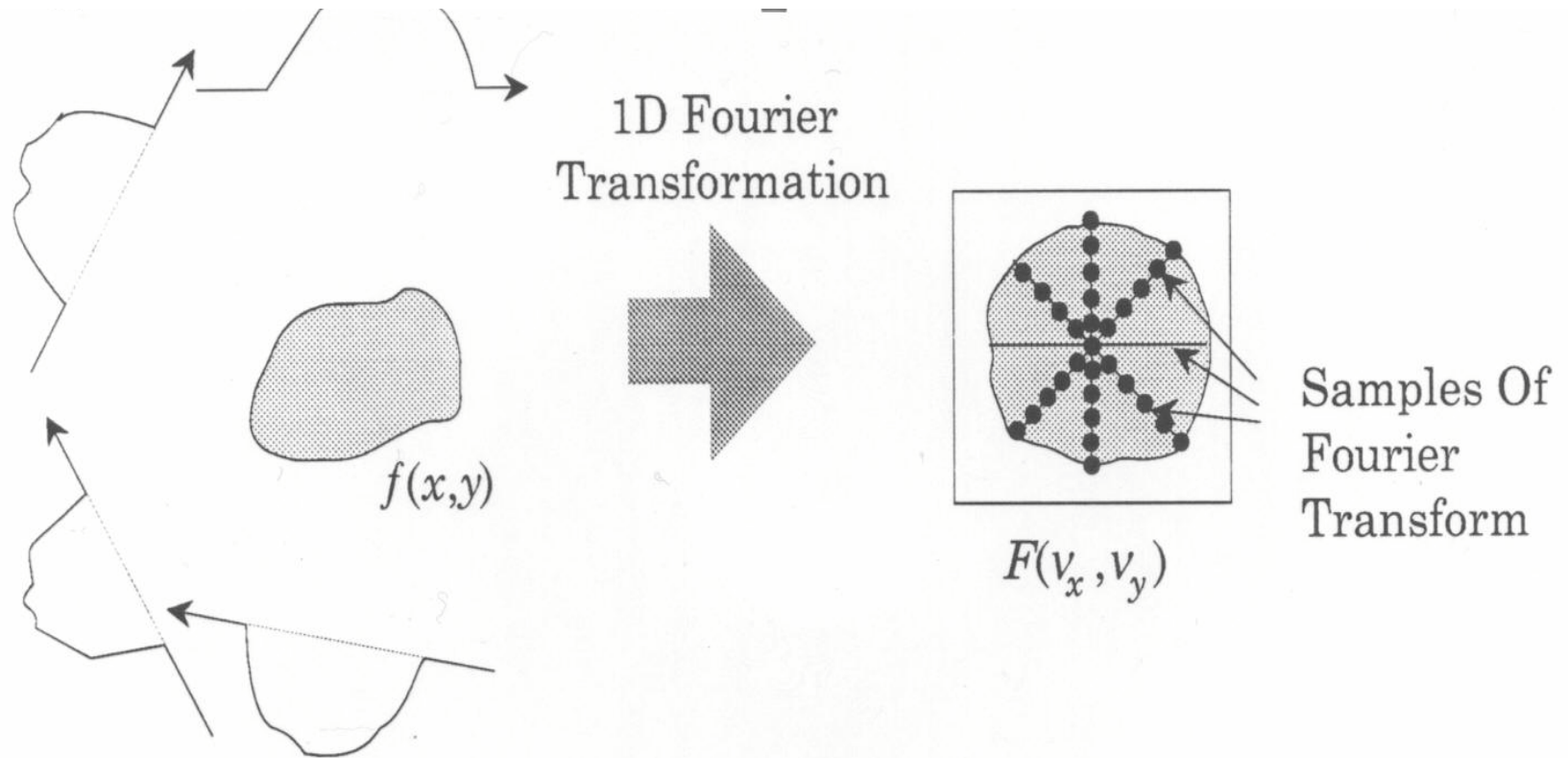
$$f(x, y) = \int_0^{2\pi} \int_0^{\infty} F_p(w, \theta) \cdot e^{j2\pi \cdot (x \cdot w \cdot \cos\theta + y \cdot w \cdot \sin\theta)} w \cdot dw \cdot d\theta$$

$$f(x, y) = \int_0^{\pi} \int_{-\infty}^{\infty} F_p(w, \theta) \cdot e^{j2\pi \cdot (x \cdot w \cdot \cos\theta + y \cdot w \cdot \sin\theta)} |w| \cdot dw \cdot d\theta$$

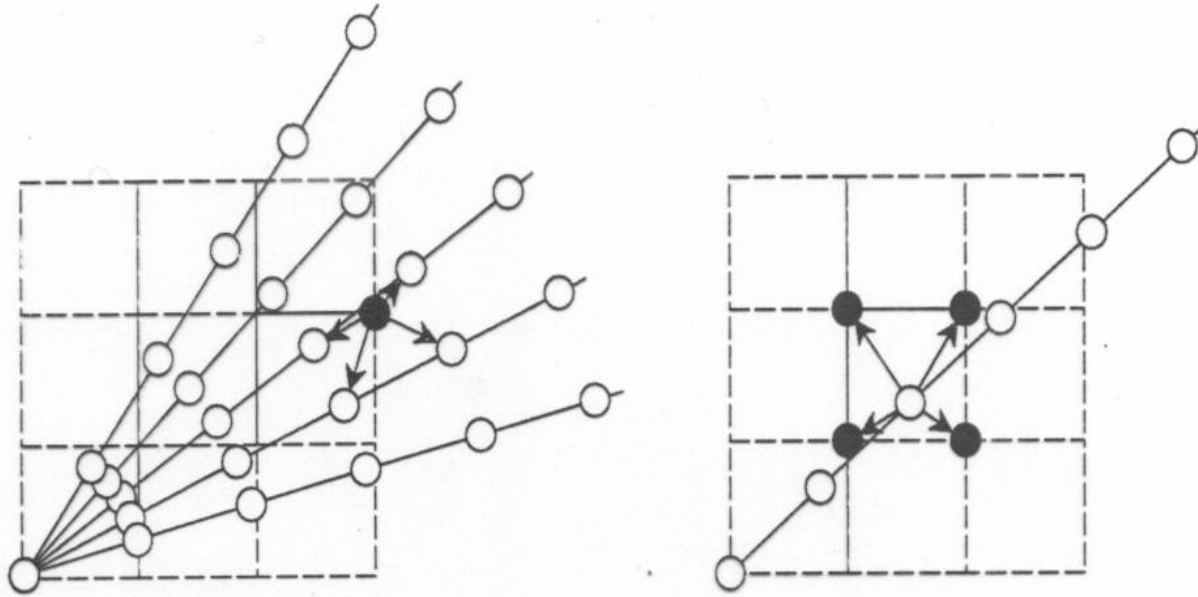
Reconstr. baseados em Transf.

- ❑ Direct Fourier Method
- ❑ Inverse Radon Transform
- ❑ Convolution Backprojection
- ❑ Filtered Backprojection
- ❑ Fan-beam
 - rebinning
 - fórmula

Ilustração DFM



DFM : interpolação em freq.

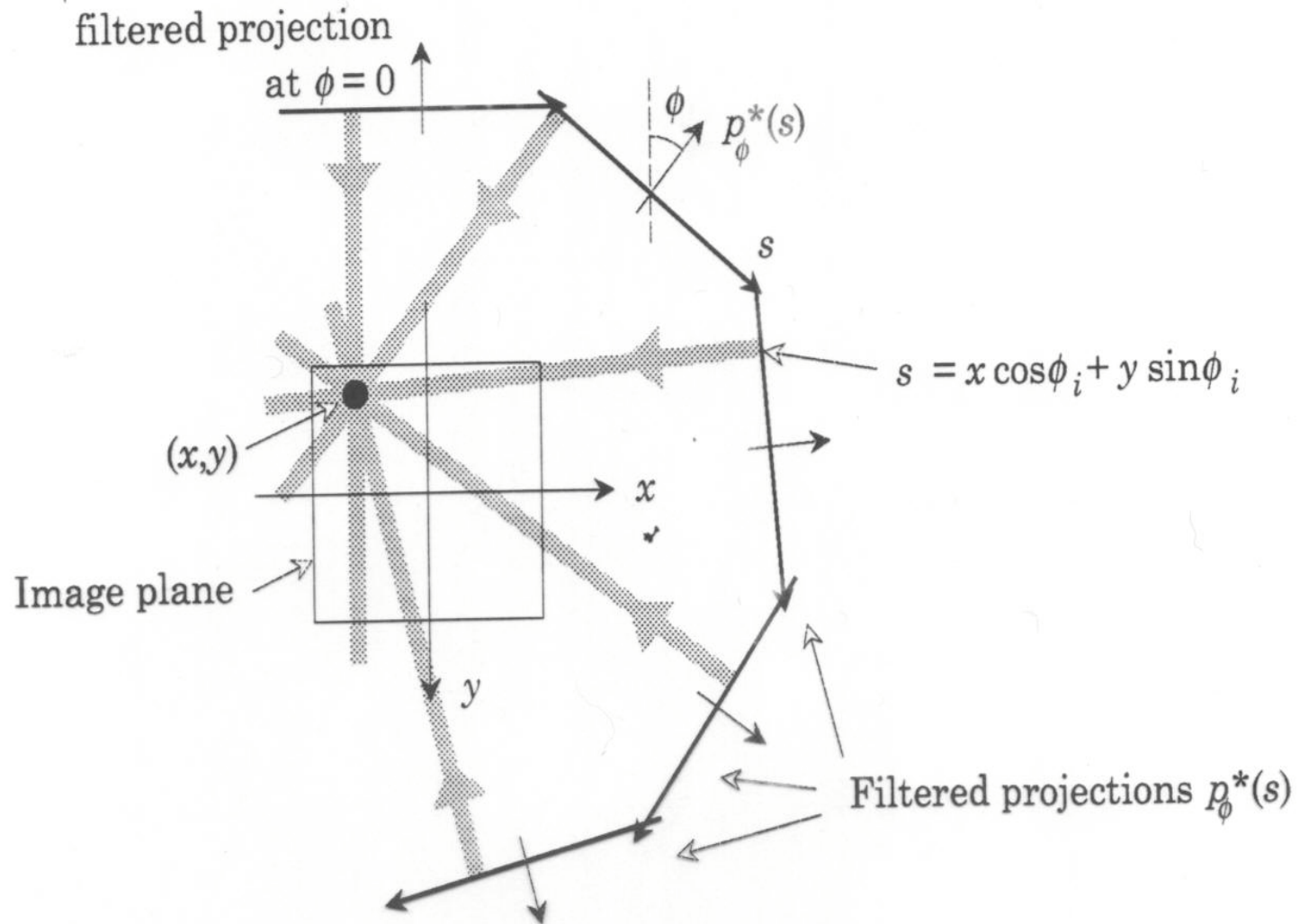


$$f(x, y) = \int_0^{\pi} \int_{-\infty}^{\infty} F_p(w, \theta) \cdot e^{j2\pi \cdot (x \cdot w \cdot \cos\theta + y \cdot w \cdot \sin\theta)} \cdot |w| \cdot dw \cdot d\theta$$

$$f(x, y) = \int_0^{\pi} \left\{ \int_{-\infty}^{\infty} F_p(w, \theta) \cdot |w| \cdot e^{j2\pi \cdot w \cdot (x \cdot \cos\theta + y \cdot \sin\theta)} dw \right\} \cdot d\theta$$

$$f(x, y) = \int_0^{\pi} \hat{g}(x \cdot \cos\theta + y \cdot \sin\theta, \theta) \cdot d\theta$$

FBP



$$f(x, y) = \int_0^{\pi} \hat{g}(x \cdot \cos \theta + y \cdot \sin \theta, \theta) \cdot d\theta$$

\Rightarrow

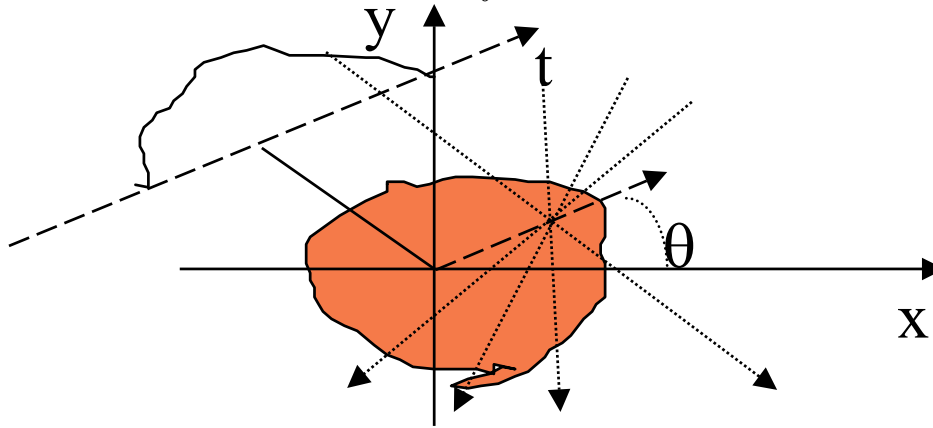
$$\hat{G}(w, \theta) = F_p(w, \theta) \cdot \text{sgn}(w) \cdot w = w F_p(w, \theta) \cdot \text{sgn}(w)$$

$$\hat{g}(s, \theta) = F^{-1}\{w F_p(w, \theta)\} \otimes F^{-1}\{\text{sgn}(w)\}$$

$$\hat{g}(s, \theta) = \left\{ \frac{1}{j2\pi} \frac{\partial g(s, \theta)}{\partial x} \right\} \otimes \left\{ \frac{-1}{j\pi \cdot s} \right\} = \frac{1}{2\pi^2} \int \frac{(\partial g / \partial t t)}{s - t} dt$$

Operador Backprojection

$$b(x, y) = B g = \int_0^{\pi} g(x \cdot \cos \theta + y \cdot \sin \theta, \theta) \cdot d\theta$$



Transformada de Hilbert de u(t)

$$h(s) = H u \equiv u(s) \otimes \frac{1}{\pi \cdot s} = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{u(t)}{s - t} dt$$

Normalização (escala Hounsfield)

$$H_{CT} = 1000 \frac{\mu - \mu_{H_2O}}{\mu_{H_2O}}$$

$$\therefore H_{CT} (\textit{água}) = 0$$

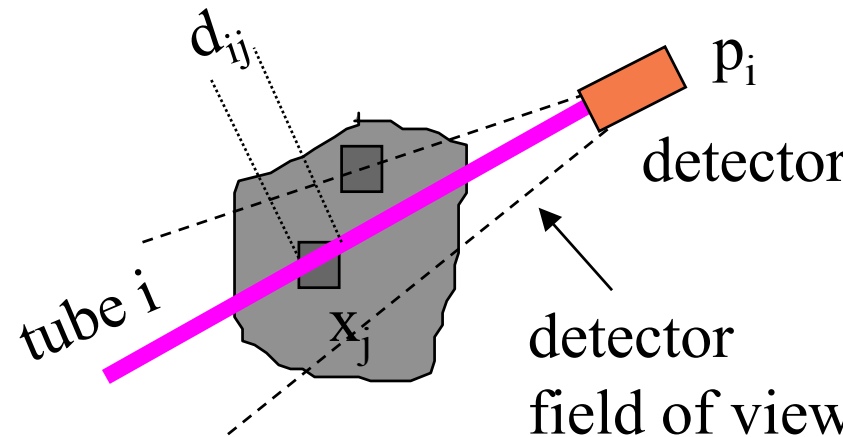
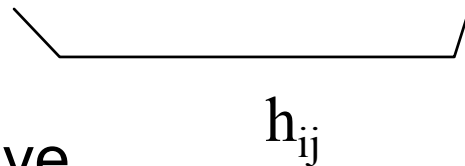
$$H_{CT} (\textit{ar}) = -1000$$

$$H_{CT} (\textit{osso}) \cong 1000$$

$$\mu_{H_2O} = 0.190 \text{ cm}^{-1} (70 \text{ keV})$$

Massa branca e cinzenta: apenas alguns Hs

$$p_i = \sum_j x_j \cdot d_{ij} \cdot A \cdot \exp\left(- \int_{s_{ij} \rightarrow \infty} \mu_j(s) ds\right)$$




Quantitative

- EM
- ART

Approximate (Transform)

- attenuation correction on projection data
- attenuation correction on reconstructed data



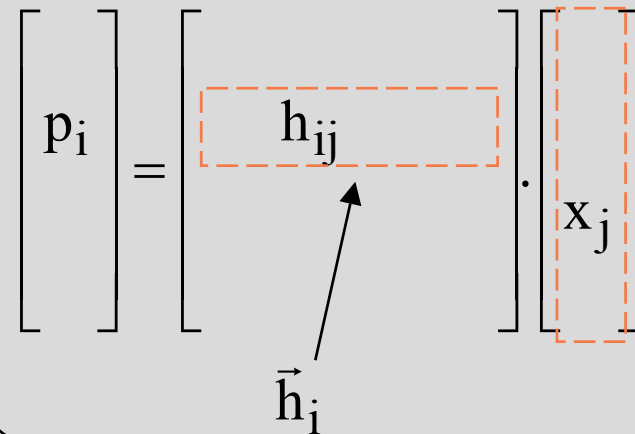
$$p_i = \sum_j h_{ij} \cdot x_j$$

Solution: Algebraic Reconstruction

- ❑ System of linear equations
 - Huge system
 - Eg. volume: 64 x 64 x 64
 x_j $j=1.. 262,144$ voxels
 - Projections:
 128 views, 64x64 planes
 p_i $i=1 .. 524,288$ projs.
 - \mathbf{H} : 524k x 262k
- ❑ Row-action methods
 - ART
 - EM

$$p_i = \sum_j h_{ij} \cdot x_j \quad (\text{all 3D projections})$$

$$\vec{p} = \mathbf{H} \cdot \vec{x} \quad (\text{vector notation})$$



$$\begin{bmatrix} p_i \end{bmatrix} = \begin{bmatrix} h_{ij} \end{bmatrix} \cdot \begin{bmatrix} x_j \end{bmatrix}$$

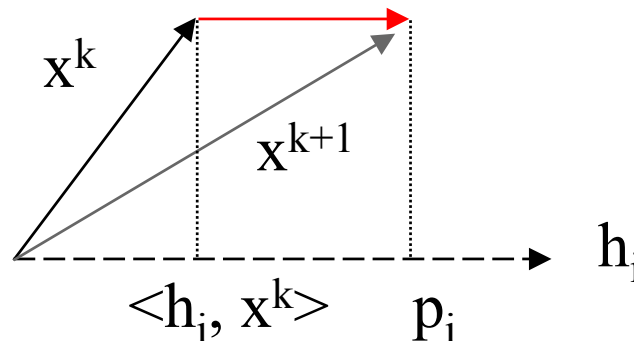
\vec{h}_i

ART: Algebraic Reconst. Technique

- ❑ Noisy data
- ❑ Optimization criteria
 - Least-square solution
 - Minimum norm solution
 - row-action
 - relaxation

$$\vec{p} = \mathbf{H} \cdot \vec{x} + \vec{n}$$

$$\vec{x}^{k+1} = \vec{x}^k + \lambda \cdot \frac{p_i - \langle \vec{h}_i, \vec{x}^k \rangle}{\|\vec{h}_i\|^2} \cdot \vec{h}_i$$



- ❑ Noise removal: projection data estimation
- ❑ Quantitative reconstruction
- ❑ Fast (3D)
- ❑ Simple
- ❑ General
- ❑ $[H]$ determination
- ❑ Stop criteria

Statistical Solution

- ❑ Projection: Poisson noise
- ❑ Maximum likelihood
 - Expectation-maximization algorithm
 - Iterative approach
- ❑ Maximum a posteriori
 - “a priori” probability distr.

$$\vec{p} = \text{Poisson}(\mathbf{H} \cdot \vec{x})$$

$$\max_{\vec{x}} \text{Pr ob}[\vec{p} | \vec{x}] \quad (\text{ML})$$

$$\max_{\vec{x}} \text{Pr ob}[\vec{x} | \vec{p}] \quad (\text{MAP})$$

$$\text{Pr ob}[\vec{x} | \vec{p}] = \frac{\text{Pr ob}[\vec{p} | \vec{x}] \cdot \text{Pr ob}[\vec{x}]}{\text{Pr ob}[\vec{p}]}$$

Expectation-maximization

- ❑ Maximum Likelihood
- ❑ ML-EM algorithm

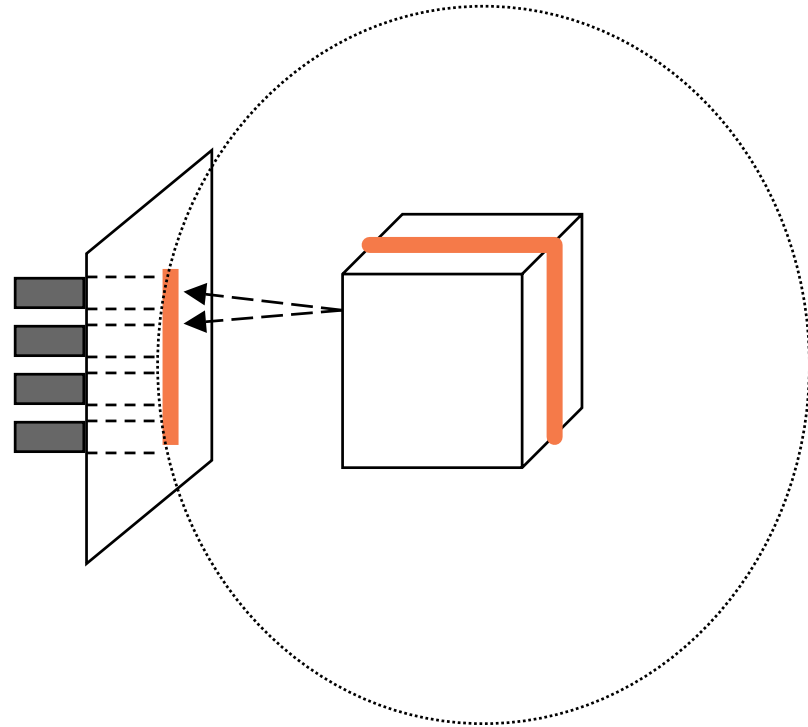
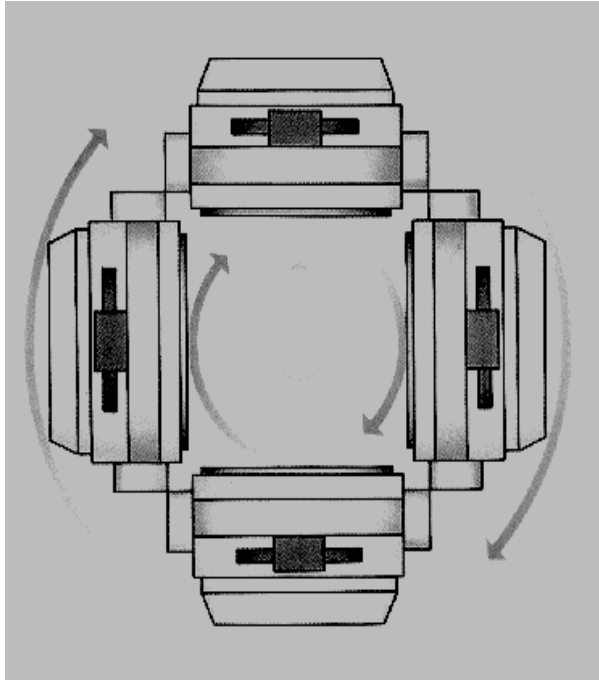
$$\max_{\vec{x}} \text{Pr ob}[\vec{p} | \vec{x}]$$

$\text{Pr ob}[\vec{p} | \vec{x}]$: independent Poisson

$$x_j^{k+1} = \frac{x_j^k}{\sum_i h_{ij}} \cdot \sum_i \frac{p_i}{\langle \vec{x}^k, \vec{h}_i \rangle} h_{ij}$$

- ❑ Handles Poisson noise
- ❑ Total count conservation
- ❑ Convergence to ML
- ❑ Expectation-maximization
 - algorithm independent of rays direction
 - quantitative approach
 - iterative
 - slow convergence
 - no stop criterion

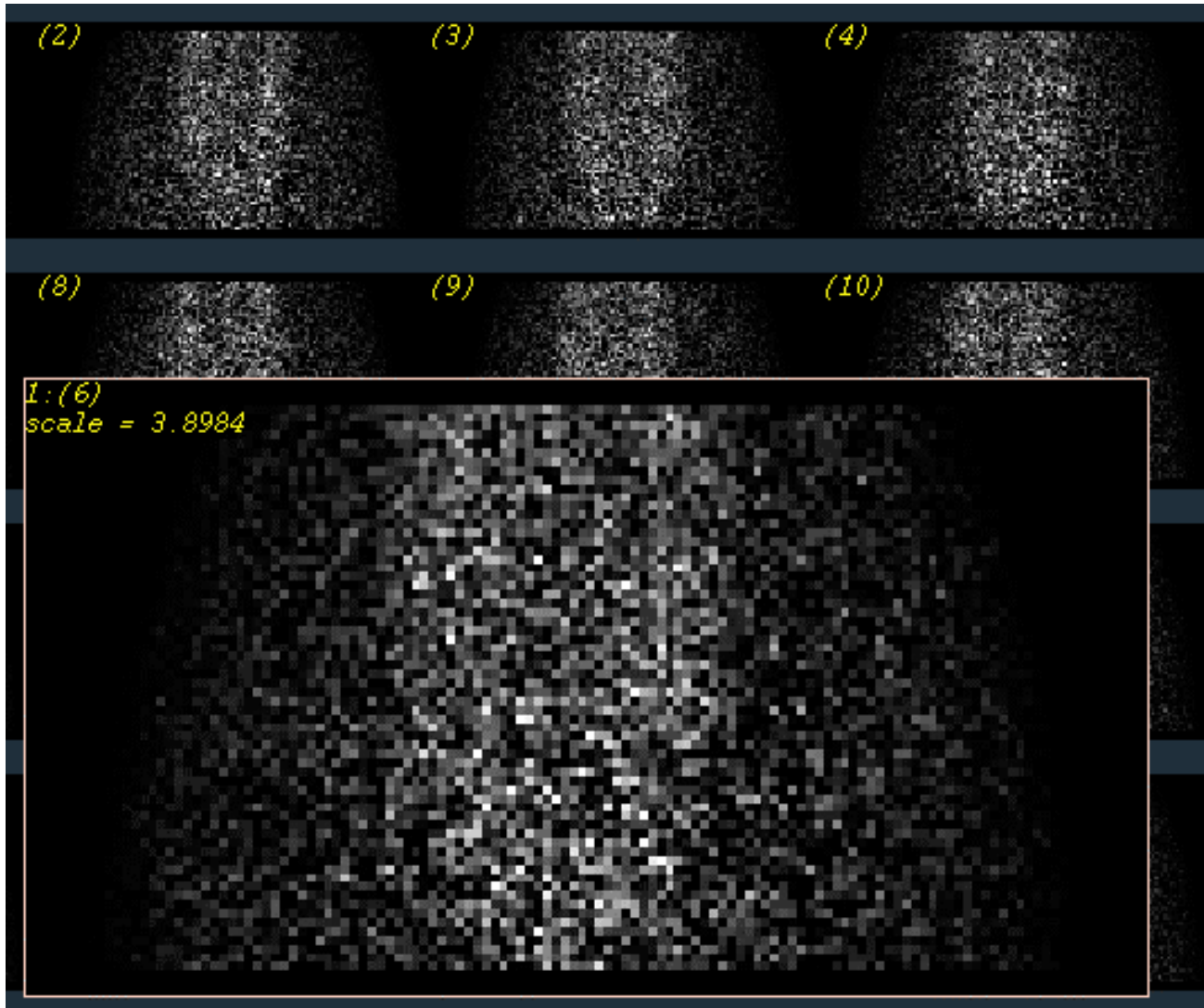
SPECT: Single Photon Emission CT



Conventional SPECT

- parallel collimator
- 2D reconstruction (slice-by-slice)

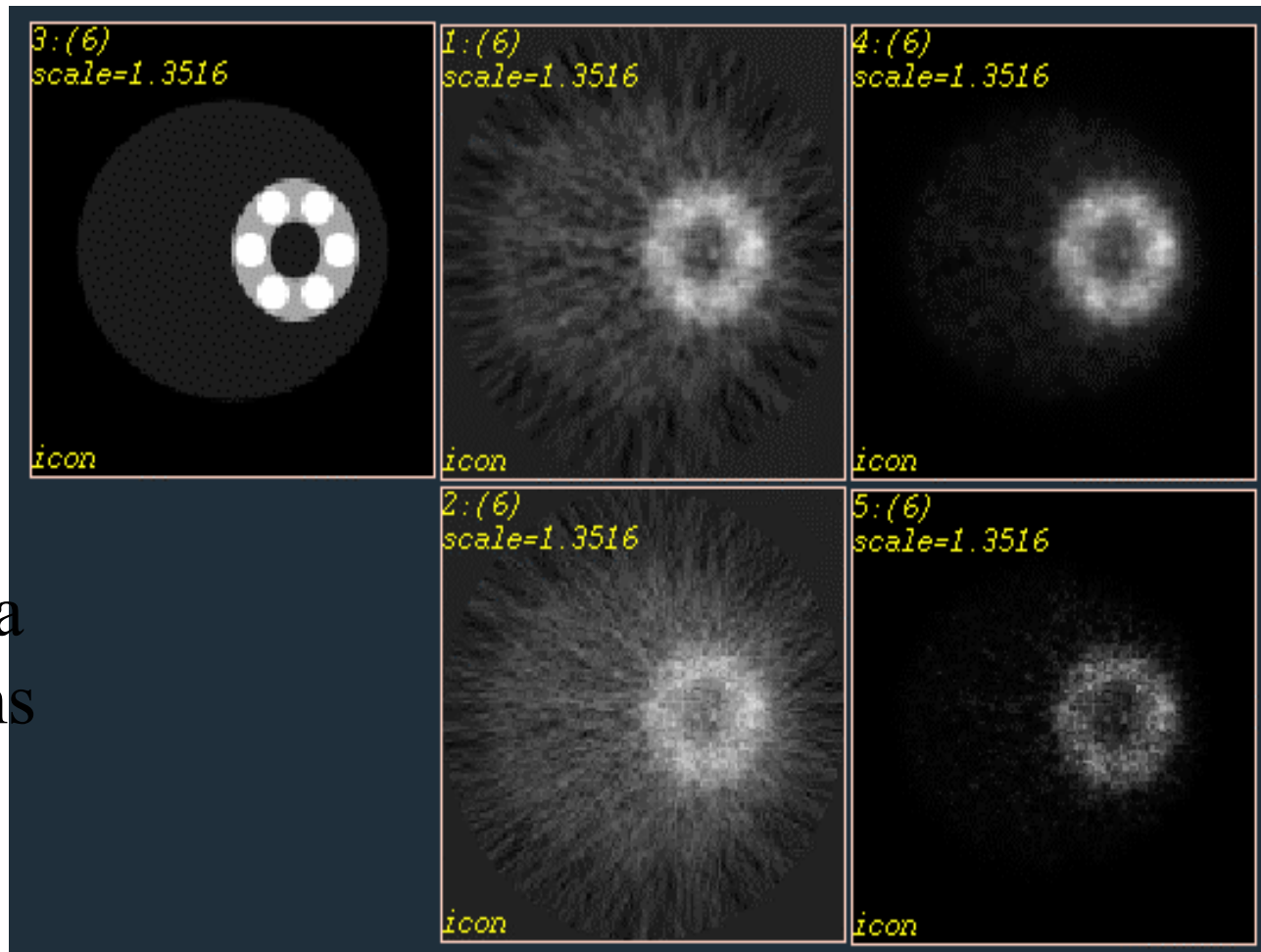
Noisy projections



Reconstructions

ART Blob

EM Blob



- Noisy data
- 5 iterations

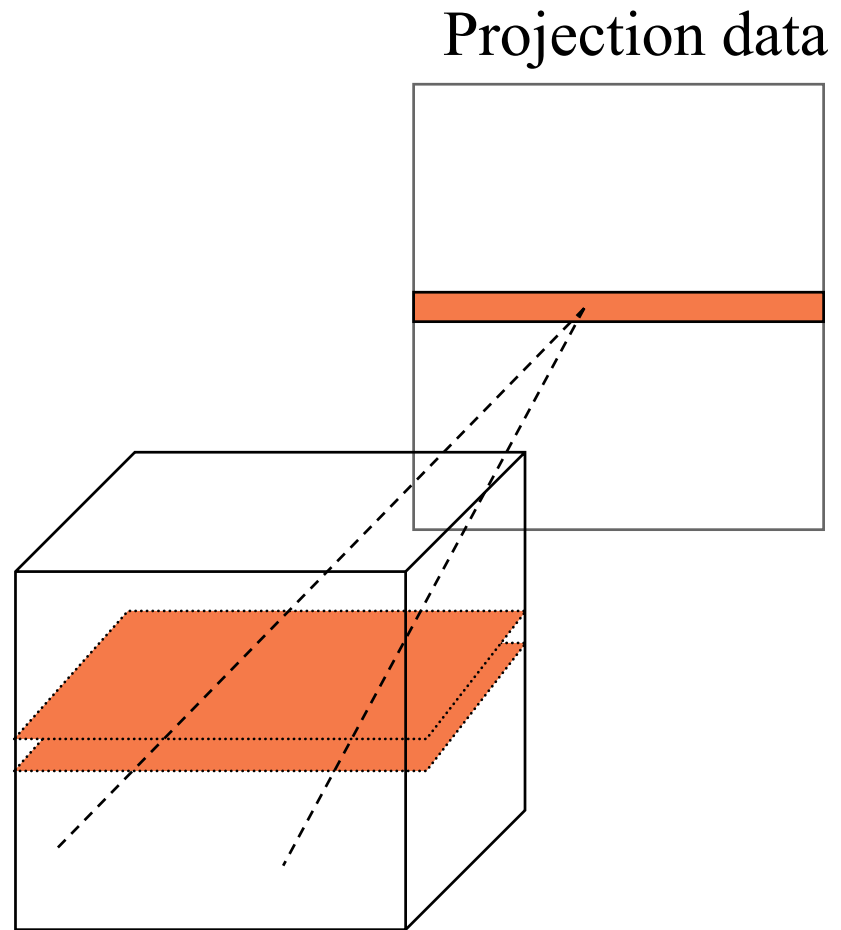
ART Voxel

EM Voxel

SPECT : parallel collimator

- ❑ Considering 2D effects
 - PSF
 - scattering
- ❑ Reconstruction approaches, slice-by-slice
 - EM
 - ART
 - 2D FBP

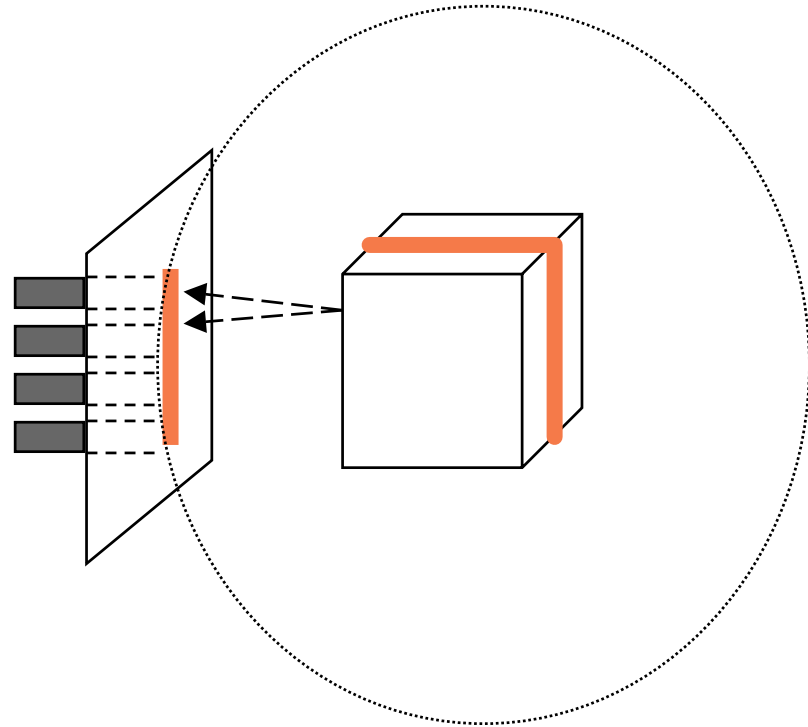
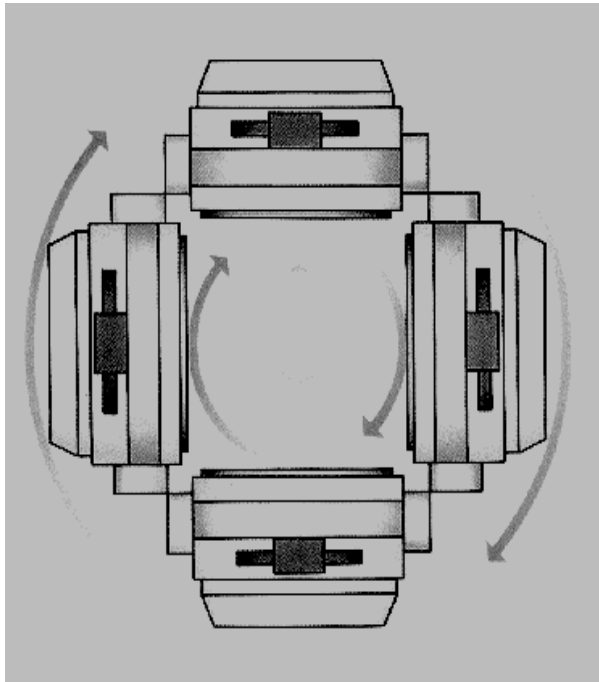
} Quantitative



3D Reconstruction in NM: outline

- ❑ Projection data formation
 - SPECT
 - 3D PET
- ❑ 3D tomographic reconstruction in NM
 - ML-EM : Maximum-likelihood
 - ART : Algebraic Reconstruction Technique
 - FBP : Filtered Backprojection
 - DFM : Direct Fourier Method
- ❑ 3D analysis
 - rendering
 - measurement

SPECT: Single Photon Emission CT



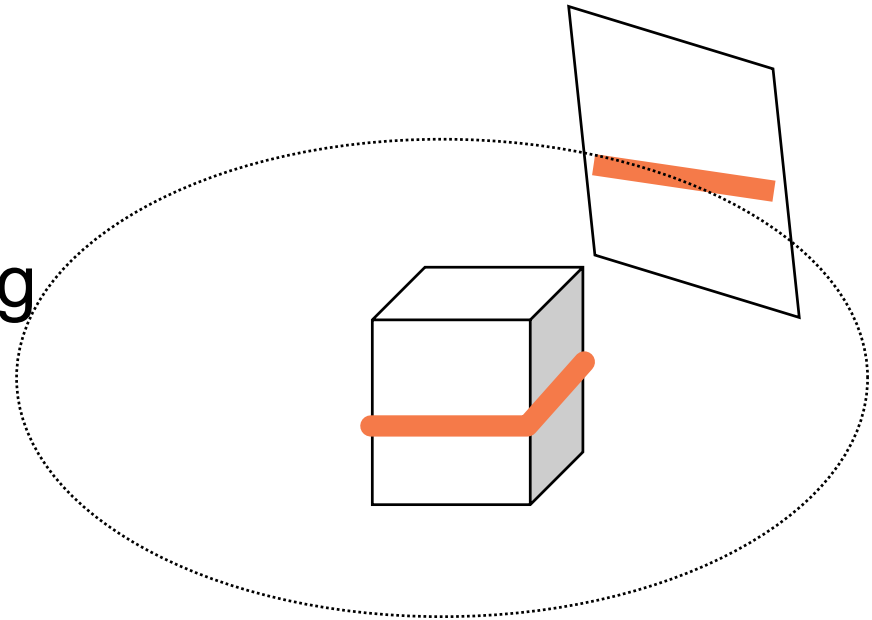
Conventional SPECT

- parallel collimator
- 2D reconstruction (slice-by-slice)

SPECT: projection data formation

❑ Parallel Collimators

- stack of 2D slices
- 3D effects of scattering
- 3D effects of PSF



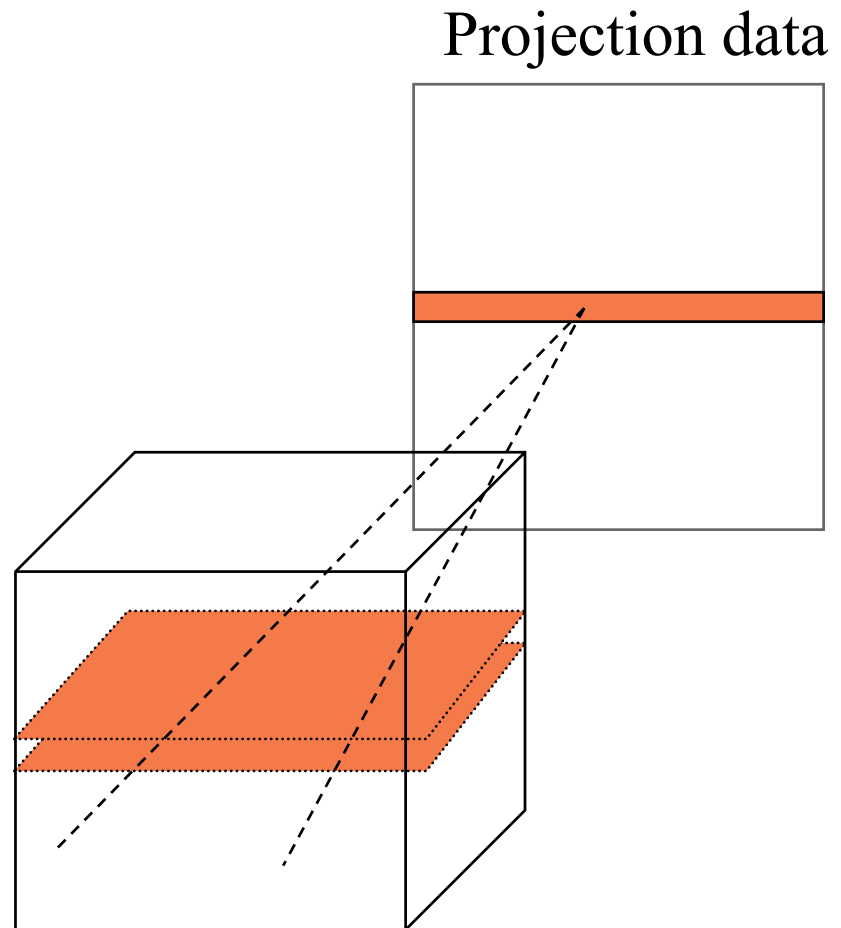
❑ Cone Beam collimators

- 3D reconstruction (3D FBP)
- improved sensitivity

SPECT : parallel collimator

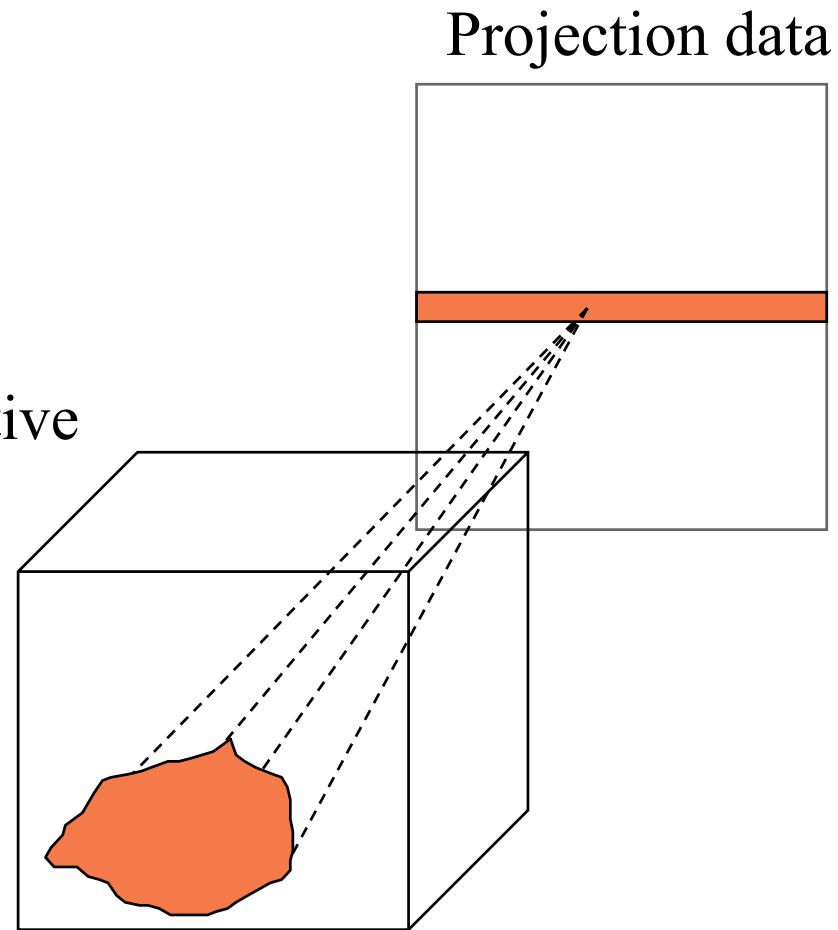
- ❑ Considering 2D effects
 - PSF
 - scattering
- ❑ Reconstruction approaches, slice-by-slice
 - EM
 - ART
 - 2D FBP

} Quantitative



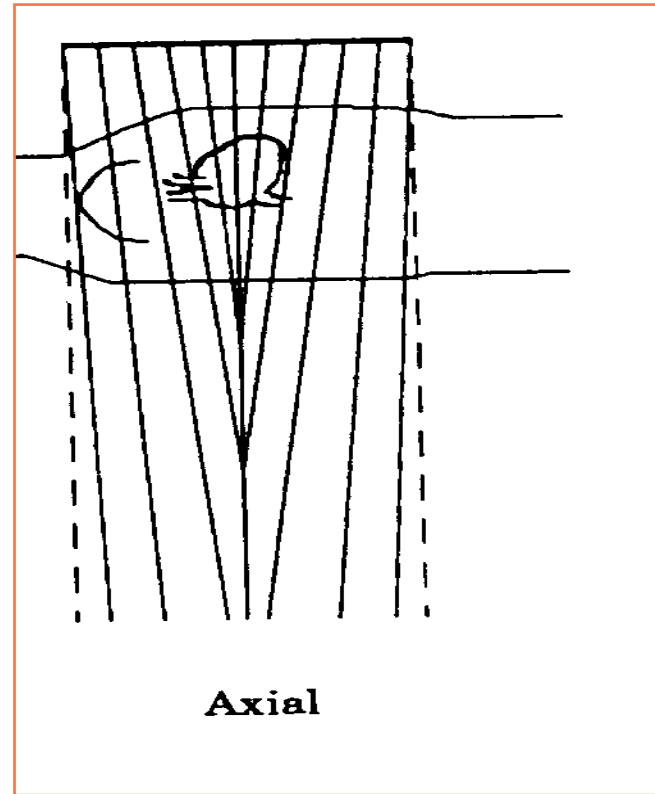
SPECT: parallel collimators

- ❑ Considering 3D effects
 - PSF
 - scattering
 - ❑ Reconstruction
 - 3D EM
 - 3D ART
 - 2D FBP with 3D filter (approximate)
- } Quantitative



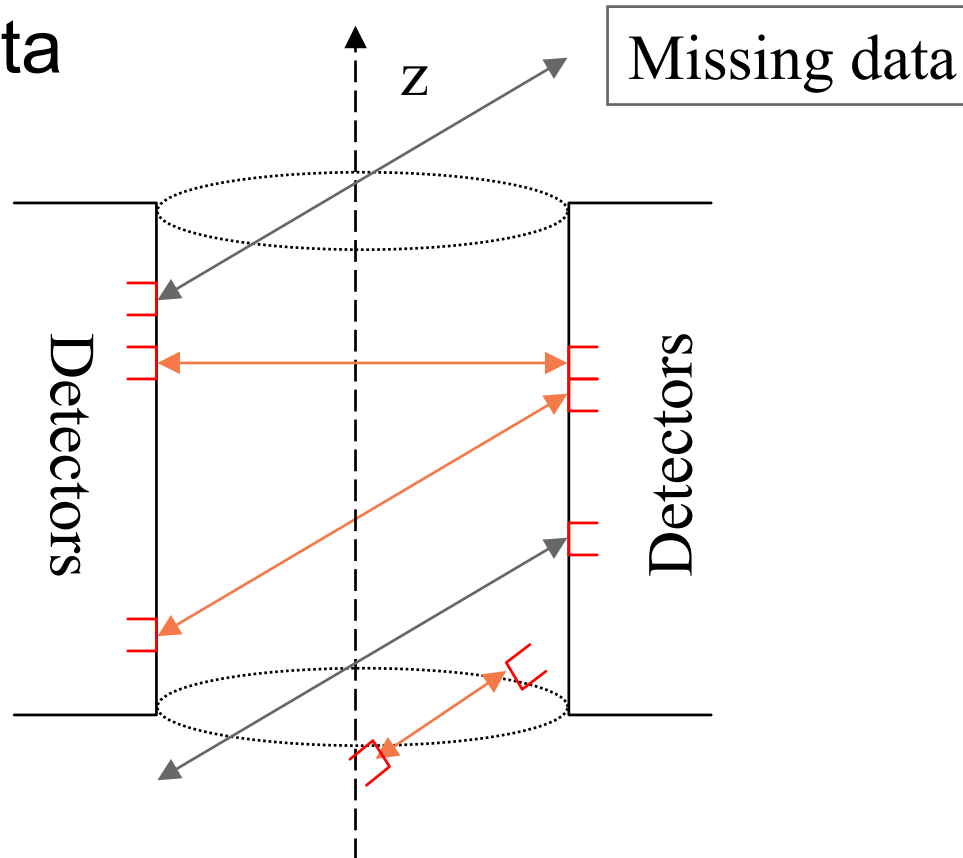
SPECT: cone beam

- uncomplete sampling => distortion
- ❑ Variable focus
 - Better Sensitivity
 - 3D FBP
 - Modified Feldkamp



3D PET: Positron Emission Tomogr.

- ❑ Transversal and tilted lines
- ❑ Missing data



Projection data formation model (3D)

□ General case (emission)

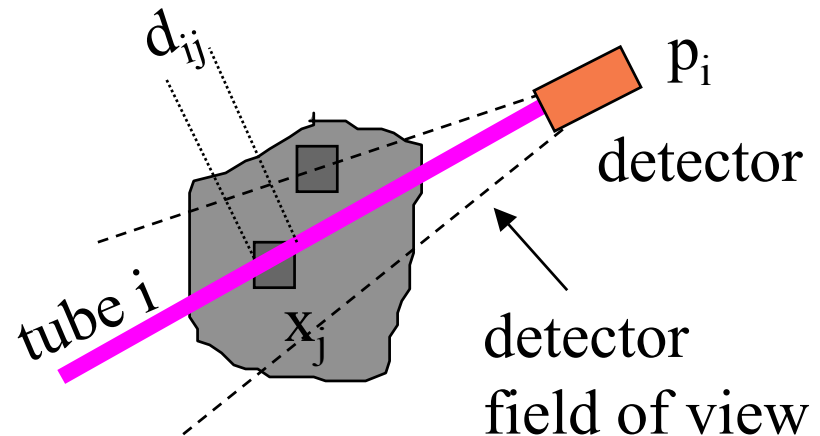
$$p_i = \sum_j x_j \cdot \underbrace{(A \cdot d_{ij})}_{\text{vol}_j} \cdot \underbrace{\exp\left(-\int \mu_j(s) ds\right)}_{\text{attenuation}}$$

x_j : emission rate per unit volume

A : area of tube cross section

d_{ij} : intersection length

$\mu_j(s)$: attenuation function



Projection data formation model (3D)

General case (emission)

$$p_i = \sum_j h_{ij} \cdot x_j$$

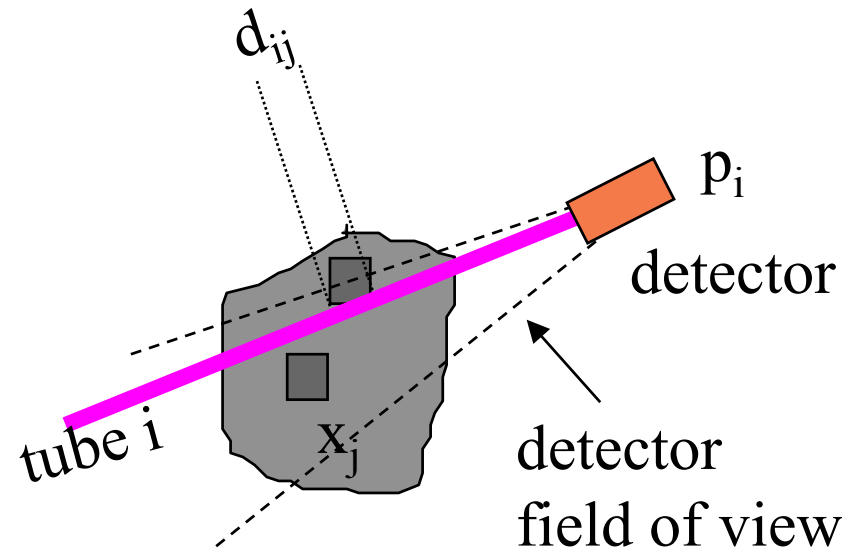
x_j : emission rate per unit volume

h_{ij} : projection coefficient

$$h_{ij} = (A \cdot d_{ij}) \cdot \exp\left(-\int_{s_j} \mu_j(s) ds\right)$$

h_{ij} encompasses:

- attenuation
- detector response
- scattering



Projection data formation model (3D)

❑ Realistic model

- attenuation
- scattering
- spatially variant PSF (Point Spread Function)

❑ Difficulties

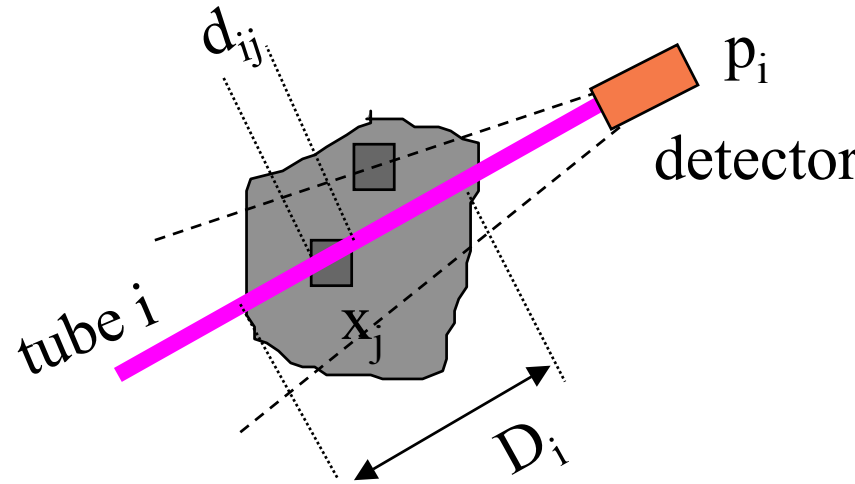
- h_{ij} determination (SPECT)
- reconstruction method : slow

Emission CT (3D)

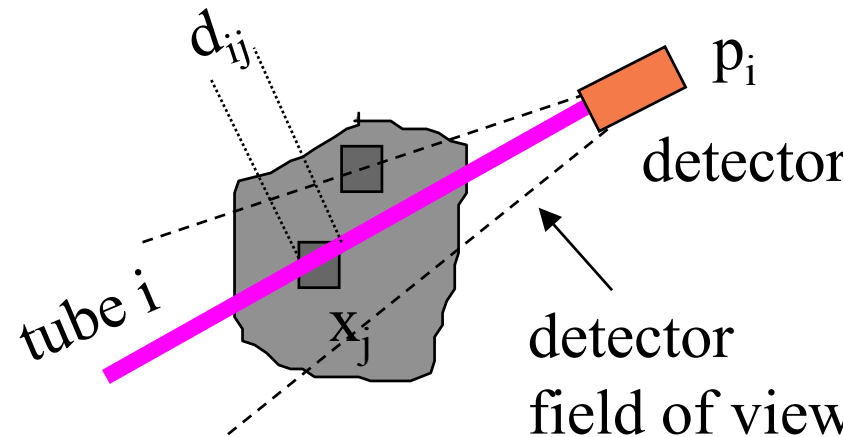
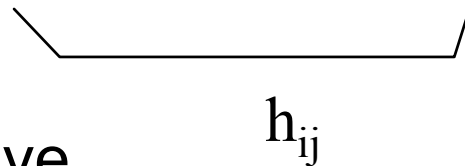
$$p_i = \sum_j x_j \cdot (A \cdot d_{ij}) \cdot \exp\left(-\int_{s_j} \mu_j(s) ds\right)$$

3D PET (quantitative)

- scattering correction
- attenuation correction
- EM
- ART
- Transform methods



$$p_i = \sum_j x_j \cdot d_{ij} \cdot A \cdot \exp\left(- \int_{s_{ij} \rightarrow \infty} \mu_j(s) ds\right)$$



Quantitative

- EM
- ART

Approximate (Transform)

- attenuation correction on projection data
- attenuation correction on reconstructed data

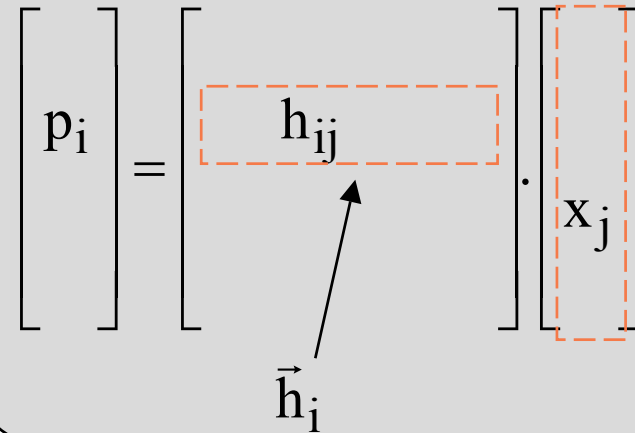
→ $p_i = \sum_j h_{ij} \cdot x_j$

Solution: Algebraic Reconstruction

- ❑ System of linear equations
 - Huge system
 - Eg. volume: 64 x 64 x 64
 x_j $j=1.. 262,144$ voxels
 - Projections:
 128 views, 64x64 planes
 p_i $i=1 .. 524,288$ projs.
 - \mathbf{H} : 524k x 262k
- ❑ Row-action methods
 - ART
 - EM

$$p_i = \sum_j h_{ij} \cdot x_j \quad (\text{all 3D projections})$$

$$\vec{p} = \mathbf{H} \cdot \vec{x} \quad (\text{vector notation})$$



$$\begin{bmatrix} p_i \end{bmatrix} = \begin{bmatrix} h_{ij} \end{bmatrix} \cdot \begin{bmatrix} x_j \end{bmatrix}$$

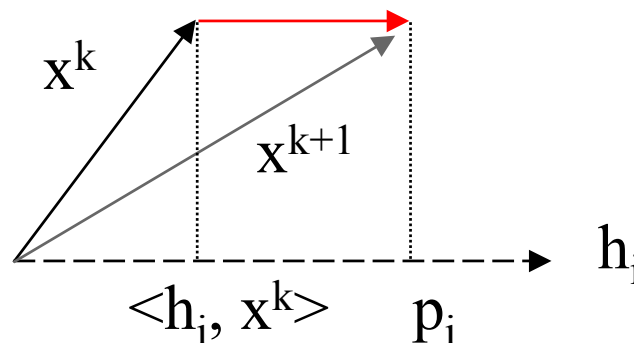
\vec{h}_i

ART: Algebraic Reconst. Technique

- ❑ Noisy data
- ❑ Optimization criteria
 - Least-square solution
 - Minimum norm solution
 - row-action
 - relaxation

$$\vec{p} = \mathbf{H} \cdot \vec{x} + \vec{n}$$

$$\vec{x}^{k+1} = \vec{x}^k + \lambda \cdot \frac{p_i - \langle \vec{h}_i, \vec{x}^k \rangle}{\|\vec{h}_i\|^2} \cdot \vec{h}_i$$



- ❑ Noise removal: projection data estimation
- ❑ Quantitative reconstruction
- ❑ Fast (3D)
- ❑ Simple
- ❑ General
- ❑ [H] determination
- ❑ Stop criteria

Statistical Solution

- ❑ Projection: Poisson noise
- ❑ Maximum likelihood
 - Expectation-maximization algorithm
 - Iterative approach
- ❑ Maximum a posteriori
 - “a priori” probability distr.

$$\vec{p} = \text{Poisson}(\mathbf{H} \cdot \vec{x})$$

$$\max_{\vec{x}} \text{Pr ob}[\vec{p} | \vec{x}] \quad (\text{ML})$$

$$\max_{\vec{x}} \text{Pr ob}[\vec{x} | \vec{p}] \quad (\text{MAP})$$

$$\text{Pr ob}[\vec{x} | \vec{p}] = \frac{\text{Pr ob}[\vec{p} | \vec{x}] \cdot \text{Pr ob}[\vec{x}]}{\text{Pr ob}[\vec{p}]}$$

Expectation-maximization

- ❑ Maximum Likelihood
- ❑ ML-EM algorithm

$$\max_{\vec{x}} \text{Pr ob}[\vec{p} | \vec{x}]$$

$\text{Pr ob}[\vec{p} | \vec{x}]$: independent Poisson

$$x_j^{k+1} = \frac{x_j^k}{\sum_i h_{ij}} \cdot \sum_i \frac{p_i}{\langle \vec{x}^k, \vec{h}_i \rangle} h_{ij}$$

- ❑ Handles Poisson noise
- ❑ Total count conservation
- ❑ Convergence to ML
- ❑ Expectation-maximization
 - algorithm independent of rays direction
 - quantitative approach
 - iterative
 - slow convergence
 - no stop criterion

Transform Methods

- ❑ Based on line integrals
- ❑ Well behaved geometry (Parallel rays, cone-beam,)
- ❑ Assuming data corrected for
 - Attenuation
 - Scattering
 - Spatial variant PSF
- ❑ Direct Fourier Method (DFM)
- ❑ Filtered Back Projection (FBP)

3D Direct Fourier Method

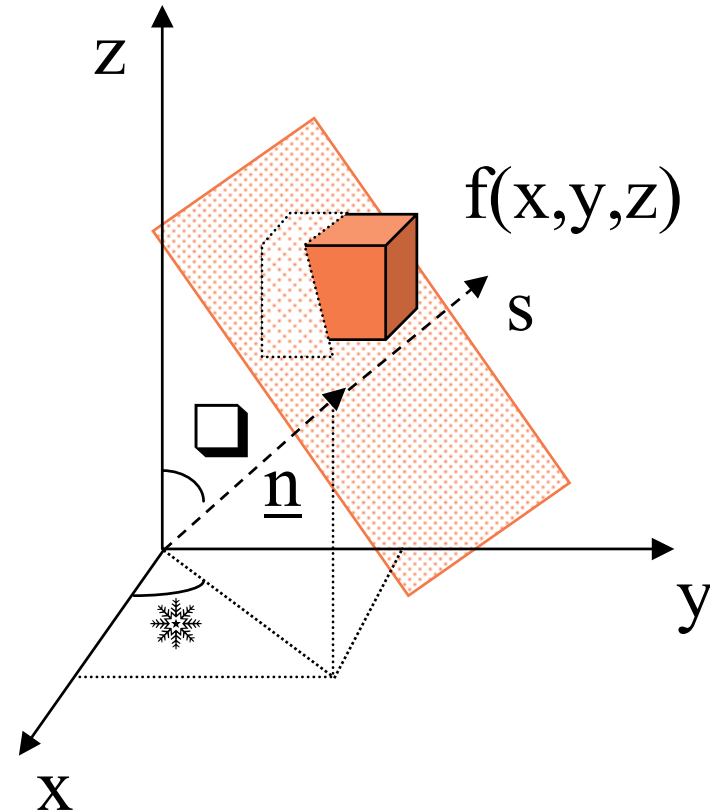
3D Radon Transform

$$g(s, \vec{n}) = \iiint f(\vec{r}) \cdot \delta(\vec{r} \cdot \vec{n} - s) d\vec{r}$$

$$G(w, \vec{n}) = F(w \cdot \vec{n})$$

3D projection theorem

- Estimation of 3D grid in Frequency domain
- 3D inverse Fourier Transf.



Filtered Back Projection

- ❑ Slice-by-slice FBP (2D)
- ❑ Filtering 2D projection
 - 3D Back Projection
- ❑ 3D Inverse Radon
- ❑ Cone-beam=> Grangeat
 - 2-stages 2D reconstr.

$$f(\vec{r}) = \int_0^\pi \int_0^{2\pi} \hat{g}(\vec{r} \cdot \vec{n}, \vec{n}) \cdot \sin \theta \cdot d\phi d\theta$$

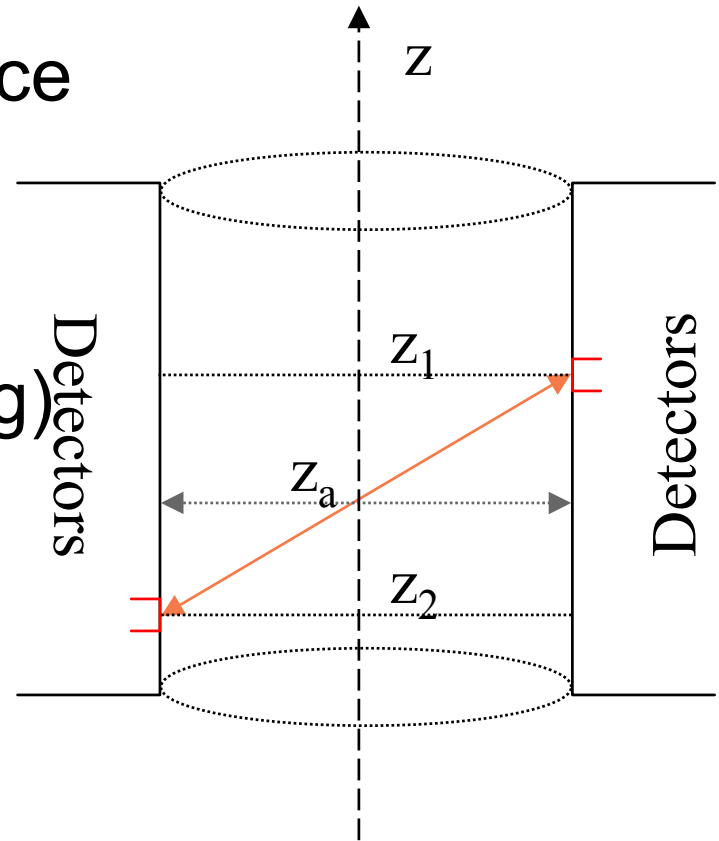
$$\hat{g}(s, \vec{n}) = \frac{-1}{8\pi^2} \frac{\partial^2 g(s, \vec{n})}{\partial s^2}$$

Projection with missing data

- Stack of 2D slices
- Single-slice Rebinning
- Multiple-slice Rebinning
- 3D Reprojection Method
- 3D Direct Fourier Method
- EM
- ART

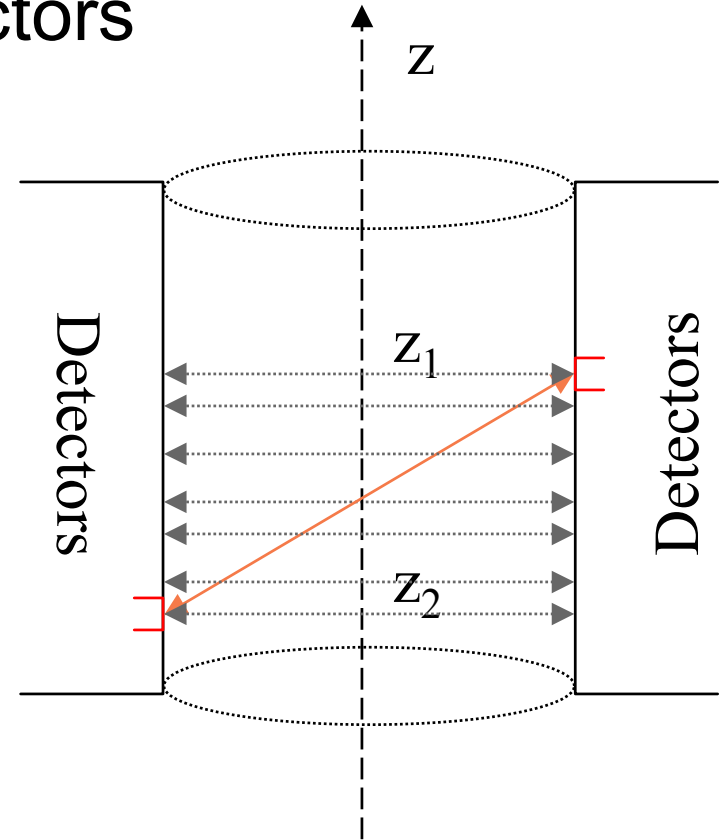
Single-slice rebinning

- ❑ 3D PET: multiple rings of detectors
 - Detection: intermediate slice
 - 2D FBP (slice-by-slice)
 - fast reconstruction
 - blur (axial aperture > 9 deg)
 - loss of resolution



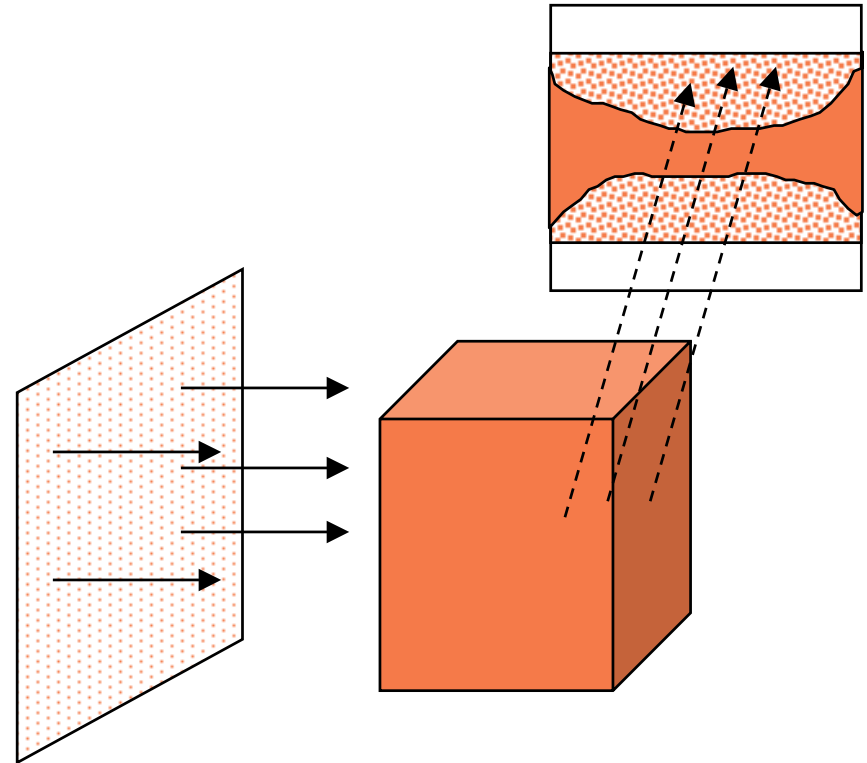
Multiple-slice rebinning

- ❑ 3D PET: multiple rings of detectors
 - detection: distributed along intermediate slices
 - deblurring along z-axis
 - quantitative approach



3D Reprojection

- ❑ Missing data estimation
- ❑ Filtering
- ❑ 3D back projection



3D Reprojection method

- ❑ Missing data
- ❑ Rebinning to parallel rays (tilted planes)
- ❑ Scattering and attenuation correction
- ❑ Reconstruction
 - Initial slice-by-slice reconstruction using transversal projections(FBP)
 - Forward projection => missing data estimation
 - Colsher's filter (2D filter)
 - 3D backprojection

SPECT: reconstruction

- ❑ EM
- ❑ ART
- ❑ Transform methods problems:
 - 3D scattering
 - 3D PSF effects
 - attenuation

SPECT: transform methods

- ❑ stack of 2D reconstructions
 - attenuation correction: Chang
 - PSF: 2D Metz filter
 - poor quantitative results
- ❑ 3D compensation
 - 2D reconstructions, with attenuation correction
 - PSF: 3D deconvolution filter
 - poor quantitative results

□ Fourier coefficients

- matched filter
- transformed basis function

$$f(\vec{r}) = \sum_{k=1}^{\infty} F_k \cdot \Phi_k(\vec{r})$$

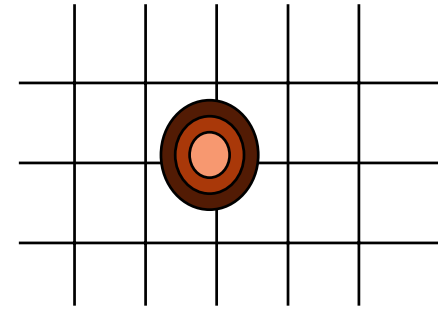
$$g = H \cdot f = \sum_{k=1}^{\infty} F_k \cdot H\Phi_k(\vec{r})$$

$$\hat{F}_k = \sum_{m=1}^M (H\Phi_k)^* \cdot g_m$$

$$\hat{F}_k = h^{(k)} \cdot g$$

Blobs: ART, EM

- ❑ Spherically symmetric basis function
- ❑ generalized Kaiser-Bessel function



$$b_{m,a,\alpha}(r) = \left[\sqrt{1 - \left(\frac{r}{a}\right)^2} \right]^m \cdot \frac{I_m \left(d \cdot \sqrt{1 - \left(\frac{r}{a}\right)^2} \right)}{I_m(\alpha)}$$

r: radial distance ($r \leq a$)

I_m : modif. Bessel function of order m

a: blobs radius

α : parameter

3D analysis

- ❑ Assembly
- ❑ Rendering
 - Surface
 - Volume
- ❑ Measurement

Summary

- ❑ 2D/3D reconstruction
- ❑ Reconstruction time
- ❑ Reconstruction quality
- ❑ Quantitative SPECT/PET

❑ Objetivos:

- otimização do processo de reconstrução em SPECT (qualidade e velocidade)
- infraestrutura flexível p/ avaliação objetiva de novos procedimentos e métodos
- estudo de técnicas alternativas para reconstrução tomográfica

❑ Motivação:

- Sistema integrado com reconstrução, manipulação, quantificação e análise 3D;
- Plataforma mais flexível para pesquisa de alternativas em reconstrução;

- ❑ Reconstrução 2D a partir de projeções 1D
- ❑ Empilhamento de “slices” => estruturas 3D
- ❑ Técnicas de reconstrução
 - FBP (Filtered Backprojection) com filtros
 - DFM (Direct Fourier Method)
 - ART (Algebraic Reconstruction Tech.)
 - EM (Expectation - Maximization)
 - Estimativa das projeções e FBP
- ❑ Otimização dos algoritmos: especificidades do SPECT
 - plano de projeções transversais independentes
 - ruído Poisson
 - ART

□ Avaliação

- phantoms sintéticos com parâmetros randômicos
 - dependente do modelo da formação
 - boa comparação qualitativa (estatística)
 - dependente da definição da figura-de-mérito
- phantoms físicos
 - limitada avaliação quantitativa
 - boa avaliação qualitativa
 - limitados casos e situações
- pacientes
 - subjetiva

Dificuldades

- efeito da atenuação em estruturas complexas como o coração
- efeito do movimento, mesmo no gated
- definição de figura-de-mérito apropriada

- ❑ Algoritmo da estimativa das proj.
 - implantado no PC, VMS e SUN (AVS)
 - *a implementar: otimização (versão AVS)*
- ❑ DFM
 - implantado no PC, VMS e SUN (AVS)
 - *a implementar: otimização (versão AVS)*
- ❑ FBP
 - implantado no PC
 - *a implementar: versão AVS*
 - *a implementar: otimização*
- ❑ ART e EM
 - implantado na SUN p/ caso 3D PET
 - *a implementar: adaptação p/ 2D*
- ❑ Avaliação (Geração de Phantoms, Figura-de-mérito, comparação estatística)
 - implantado na SUN p/ caso 3D PET
 - *a implementar: adaptação p/ 2D*

- ❑ Caracterização do ruído nas projeções
 - phantom físico, cilíndrico, centrado
 - obtenção do perfil com pouco ruído (média de grande número de proj.)
 - cálculo da variancia nas proj.
 - modelar ruído nos dados observados, levando-se em conta diferentes níveis de contagem
 - aplicar correções (atenuação, espalhamento, uniformidade)
 - modelar ruído após as correções
 - extrapolar modelo p/ objetos não homogêneos. Testar validade.

Reconstrução Tomográfica Tridimensional em Medicina

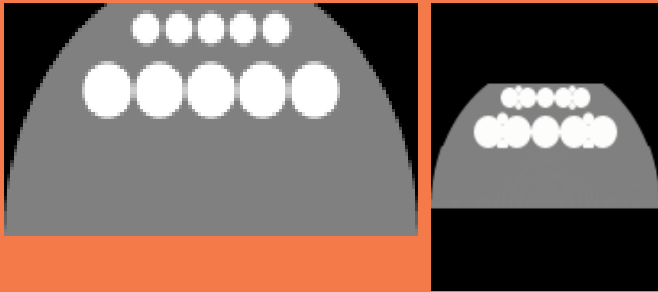
Sérgio S. Furuie, Matthias L. Egger
Divisão de Informática
Instituto do Coração - HC.FMUSP

- ❑ Modelos de formação das projeções
- ❑ Reconstrução tomográfica 2D/3D
 - Aspectos gerais
 - Vantagens/desvantagens
- ❑ Métodos de reconstrução
 - Algébrico (ART)
 - Estatístico (ML-EM)
 - Analíticos
 - FBP, DFM
 - 3DRP, Favor, Cone
 - Simplificações: Rebinning

- ❑ SPECT/PET 3D quantitativo em tempo clínico aceitável
 - Algoritmos mais eficientes
 - Otimização de todo o processo
 - Avaliação objetiva de cada fase do processo
 - Avaliação clínica
- ❑ Correção de atenuação em PET e SPECT sem fonte externa
- ❑ PET 3D dinâmico
- ❑ Dinâmica metabólica

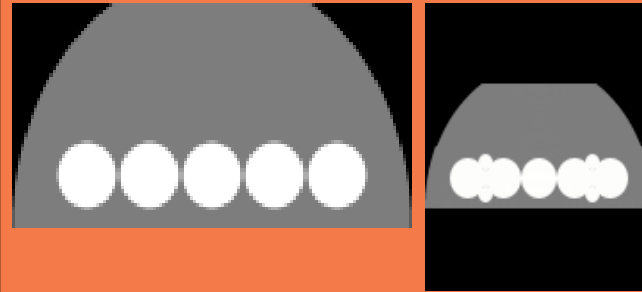
Phantoms 3D de ativ. e aten.

Atividade



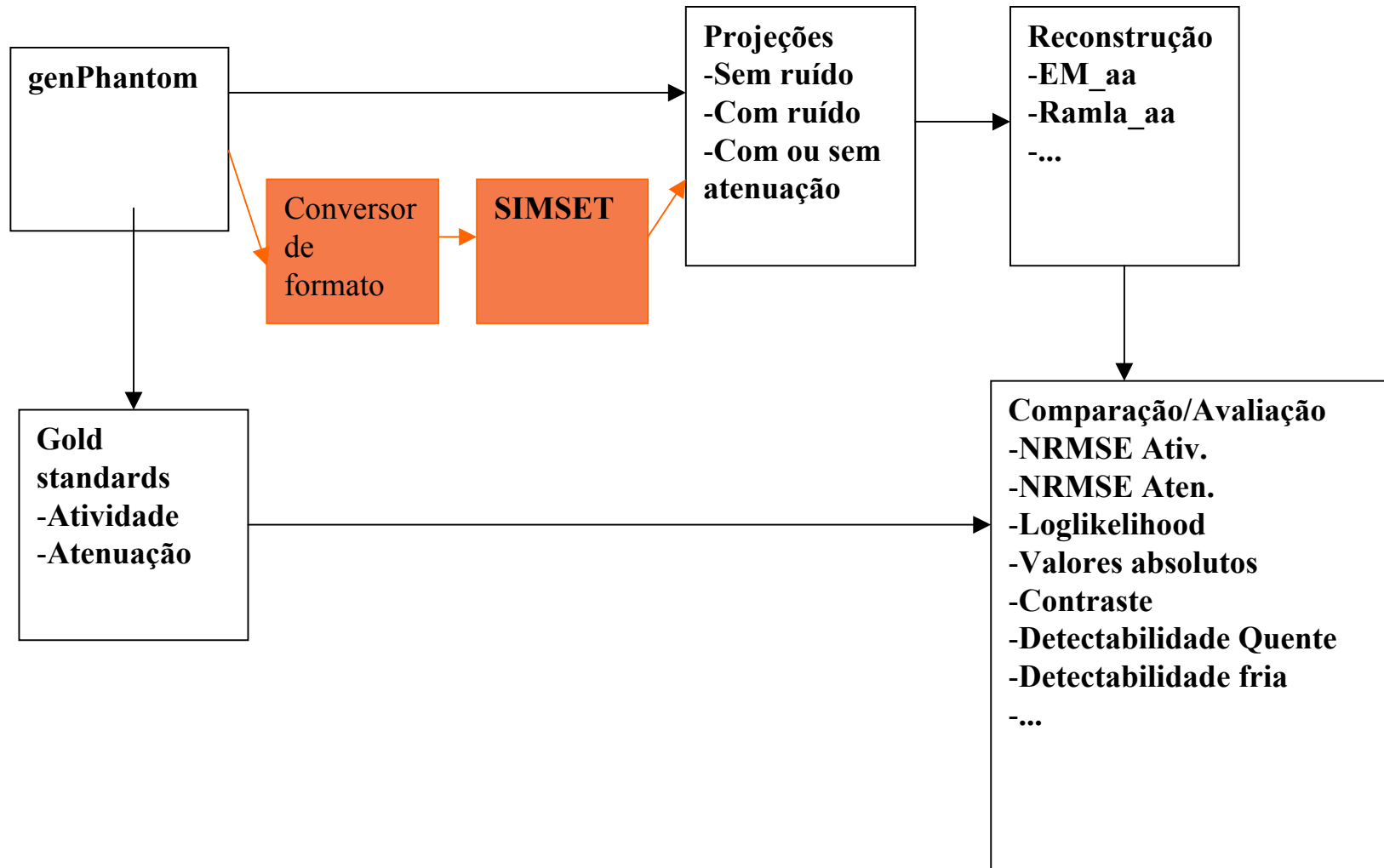
Corte em z render3D

Atenuação



Corte em z render3D

SIMSET: ruído realista



Acesso aos fontes do SimSET

University of Washington

Division of Nuclear Medicine

SimSET
Home Page



<http://depts.washington.edu/~simset>

Reconst. Tomog. a partir de projeções

- ❑ A.K. Jain, *Fundamentals of Digital Image Processing*, Prentice Hall, 1989.
- ❑ G.T. Herman, *Image Reconstruction from Projections*, Academic Press, 1980.
- ❑ J.C.Russ, *The Image Processing Handbook*, CRC Press, 1992.
- ❑ L.A. Shepp, Y.Vardi “Maximum likelihood reconstruction for emission tomography”, *IEEE Trans.Med.Imag.*, vol.1(2):113-122, 1982.

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- ❑ G.T. Herman, *Image Reconstruction from Projections*, Academic Press, 1980.
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- ❑ S.Matej, R.M.Lewitt, “Practical considerations for 3-D image reconstruction using spherically symmetric volume elements, *IEEE Trans. Med.Imag.*, vol.15(1):68-78, Feb. 1996.
- ❑ L.A. Shepp, Y.Vardi “Maximum likelihood reconstruction for emission tomography”, *IEEE Trans.Med.Imag.*, vol.1(2):113-122, 1982.